1. Consider the geometric series

\[ a + ar + ar^2 + ar^3 + ar^4 + \ldots \], or \[ a(1 + r + r^2 + r^3 + r^4 + \ldots) \]

and call its partial sums

\[ s_1 = a \quad s_2 = a + ar \quad s_3 = a + ar + ar^2 \quad s_4 = a + ar + ar^2 + ar^3 \quad \ldots \quad s_n = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1} \]
as we’ve done in class.

1A: What is the formula we derived for the nth partial sum \( s_n = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1} \)?

\[ S_n = a \left( \frac{1 - r^n}{1 - r} \right) \]

1B: Suppose \( a \neq 0 \). For what values of \( r \) does the geometric series (1) converge, and for those values of \( r \), what does the series converge to?

For \( |r| < 1 \), the series converges to \( \frac{a}{1-r} \).

1C: Consider the geometric series \( 75 - 15 + 3 - \frac{3}{5} + \frac{3}{25} - \frac{3}{125} + \frac{3}{625} - \frac{3}{3125} + \ldots \). Write it in the form \( a(1 + r + r^2 + r^3 + r^4 + \ldots) \). Clearly indicate what \( a \) and \( r \) are.

I did it in two steps: There is a 3 in each term so take that out:

\[ = 3 \left( 25 - 5 + \frac{1}{5} + \ldots \right) \]

next, take 25 out of each term:

\[ = 3 \cdot 25 \left( 1 - \frac{1}{5} + \frac{1}{125} + \ldots \right) \]

this shows \( a = 75 \) and \( r = -\frac{1}{5} \).

1D: Find the 5th partial sum \( s_5 \) of the series in 1C using the correct answer to 1A; show all your work and keep it all in terms of fractions. The answer should be written as an improper fraction in lowest terms.

Using the formula in 1A with \( n = 5 \) we get

\[ s_5 = 75 \left( 1 - \left( -\frac{1}{5} \right)^5 \right) = 75 \left( 1 + \frac{1}{3125} \right) = 75 \cdot \frac{3126}{3125} = \frac{3126}{625} \]

\[ = \frac{6 \cdot 5 \cdot 1563}{5^3 \cdot 5} = \frac{1563}{5^2} = \frac{1563}{25} \]

1E: Now add those first five terms together explicitly by your calculator. (Just enter them “as-is”, one-by-one into your calculator, and add them up). Give the answer as a decimal number to as many places as your calculator shows.

\[ 75 - 15 + 3 - \frac{3}{5} + \frac{3}{25} = 65.52 \]

1F: The answers in 1D and 1E should be the same. Verify this by converting the answer in 1D! Are they?

In 1E we get 65.52, while in 1D we get \( \frac{1563}{25} \) which the calculator says is 65.521, so yes they can equal.

1G: What does the series in 1C converge to? (Express your answer as a fraction).

\[ \frac{a}{1-r} \text{ becomes } \frac{75}{1-\left( -\frac{1}{5} \right)} = \frac{75}{\left( 5/5 \right)} = \frac{25 \cdot 5}{2} = \frac{125}{2} \]