Math 105: Review for Exam II

1. Find \( dy/dx \) for each of the following.

(a) \( y = x^2 + 2^x + e^2 + e^{2x} + \ln 2 + \ln (2x) + \arctan 2 \)

(b) \( y = \sqrt{x} \cdot \arctan (5x) \)

(c) \( y = \ln(\tan(2^{\cos(x^2)})) \)

(d) \( y = \frac{x + e^\pi}{\cos 4 + \sin^6(6x)} \)

2. Consider the curve defined by \( x^3 + y^3 = \frac{9}{2} xy \) (known as the Folium of Descartes).

(a) Find \( dy/dx \).

(b) Verify that the point (1,2) is on the curve above.

(c) Find the equation of the tangent line at the point (1,2).
3. Evaluate the following limits.

(a) \( \lim_{x \to 1} \frac{x^3 - 1}{7 - 7x} \)

(b) \( \lim_{x \to 0} \frac{1 - \cos 2x}{3x} \)

(c) \( \lim_{x \to 0} \frac{1 - \cos 4x}{5x^2} \)

(d) \( \lim_{x \to \infty} \frac{x^2}{2x} \)

4. Find the following.

(a) an antiderivative of \( y = \frac{5}{\sqrt{1 - 9x^2}} + x^3 + \cos(2x) + e^3 \)

(b) \( \tan(\arccos x) \) (rewritten as an algebraic expression - no trigonometric functions)
5. Consider the function \( f(x) = x^4e^x \) with domain all real numbers.

(a) Find the \( x \)-value(s) of all roots (\( x \)-intercepts) of \( f \).

(b) Find the \( x \)- and \( y \)-value(s) of all critical points and identify each as a local max, local min, or neither.

(c) Find the \( x \)- and \( y \)-value(s) of all global extrema and identify each as a global max or global min.

(d) Find the \( x \)-value(s) of all inflection points.

(e) Sketch \( f \).
6. How would your answers to the previous question change if the domain of $f$ were $[-10, 10]$?

7. You are planning to build a box-shaped aquarium with no top and with two square ends. Your budget is $288. If the glass for the sides costs $12 per square foot and the opaque material for the bottom costs $3 per square foot, what dimensions will maximize the volume? Be sure to show how you know you have found the maximum.