Math 105: Review for Exam II

1. Find \( dy/dx \) for each of the following.
   (a) \( y = x^2 + 2^x + e^{2x} + \ln 2 + \ln(2x) + (\ln 2)x + \arctan 2 \)

   (b) \( y = \sqrt{x} \cdot \arctan(5x) \)

   (c) \( y = \ln(\tan(2^{\cos(x^2)})) \)

   (d) \( y = \frac{x + e^\pi}{\cos 4 + \sin^5(6x)} \)

   (e) \( y = (x^2 + 1)^{\sin x} \)

2. Consider the curve defined by \( x^3 + y^3 = \frac{9}{2} xy \) (known as the Folium of Descartes).
   (a) Find \( dy/dx \).
(b) Verify that the point (1,2) is on the curve above.

(c) Find the equation of the tangent line at the point (1,2).

3. Evaluate the following limits.

(a) \[ \lim_{x \to 1} \frac{x^3 - 1}{7 - 7x} \]

(b) \[ \lim_{x \to 0} \frac{1 - \cos(2x)}{3^x} \]

(c) \[ \lim_{x \to 0} \frac{1 - \cos(4x)}{5x^2} \]

(d) \[ \lim_{x \to \infty} \frac{x^2}{2^x} \]

(e) \[ \lim_{x \to \infty} \left( 1 + \frac{4}{x} \right)^{503x} \] [Students in the 8:00 and 9:30 sections may omit this problem.]

4. Rewrite \( \tan(\arccos x) \) as an algebraic expression - no trigonometric or inverse trigonometric functions.
   [Students in the 8:00, 9:30, and 1:10 sections may omit this problem.]
5. Consider the function \( f(x) = x^4 e^x \) with domain all real numbers.

   (a) Find the \( x \)-value(s) of all roots (\( x \)-intercepts) of \( f \).
   
   (b) Find the \( x \)- and \( y \)-value(s) of all critical points and identify each as a local max, local min, or neither.

   (c) Find the \( x \)- and \( y \)-value(s) of all global extrema and identify each as a global max or global min.

   (d) Find the \( x \)-value(s) of all inflection points.

   (e) Sketch \( f \).
6. How would your answers to the previous question change if the domain of \( f \) were \([-10, 10]\)?

7. You are planning to build a box-shaped aquarium with no top and with two square ends. Your budget is $288. If the glass for the sides costs $12 per square foot and the opaque material for the bottom costs $3 per square foot, what dimensions will maximize the volume? Be sure to show how you know you have found the maximum.