There are 6 problems in this exam. On each problem, you must show all your work, or otherwise thoroughly explain your conclusions. **There is always at least one step preceding a final answer.** Units may be requested for your final answer; a point deduction will apply if they are omitted.

On the portion of the exam marked No Calculator, you will be allowed 20 minutes during which your calculator must be closed and put away. If you finish this section early, you may hand in your work early. However, **only after you hand in the “no calculators” section will you be permitted to use a calculator.**

You will have 55 minutes to complete this exam.

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<tr>
<th>Question</th>
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Problem 1-NC. (30 points) Complete five of the following six problems. Clearly cross out the remaining one.

(a) (6 points) Find the derivative of $x^2 \ln x + 2\sin x + e^\pi$.

(b) (6 points) Find the derivative of $\sqrt{\arctan \frac{x}{12} + 7}$.

(c) (6 points) Find the derivative of $\frac{e^x \sin x + e^x \cos x}{x^2}$. 
(d) (6 points) If $\sin W + \cos Y^2 = 7WY$, find the derivative $\frac{dY}{dW}$.

(e) (6 points) Find an antiderivative of $\frac{4}{\sqrt{25 - x^2}}$.

(f) (6 points) Find the derivative of $\ln\left(\ln(\ln(x + 3))\right)$.
**Problem 2-NC.** (12 points) The function $y(x)$ graphed below satisfies the differential equation

$$y' = y^{0.2} + y^{0.4}.$$  

(Do not attempt to solve this differential equation.) If $y^{-1}(x)$ is its inverse function, evaluate the derivative of $y^{-1}(x)$ at $x = 1$.

![Graph of function y(x)](image)

**Problem 3-NC.** (8 points) Simplify the expression $\sec(\arctan x)$ to eliminate any trigonometric functions.
Problem 1. (30 points) This problem concerns the implicit relationship graphed below, called a singular elliptic curve.

\[
y^2 = x^3 - 3x + 2
\]

(a) (10 points) Calculate \( \frac{dy}{dx} \) using implicit differentiation.

(b) (8 points) Find the coordinates of points A and B.

(c) (5 points) Explain how you can tell that the tangent line at \((-2, 0)\) is vertical.

(d) (7 points) What happens to \( \frac{dy}{dx} \) at \((1, 0)\)? Explain what this tells you about tangent lines.
Problem 2. (30 points) When evaluating the following limits, be sure to explain each step briefly.

(a) (15 points) Evaluate \( \lim_{x \to 0} \frac{e^{x^2} - x^2 - 1}{x^4} \).

(b) (15 points) Predict the value of \( \lim_{x \to \infty} \frac{x^{175}}{e^x} \), and explain how you can tell.
Problem 3. (40 points) Dr. Evil has decided his underground lair is too dank. He wants to install a Norman window, shaped as shown.

Of course, the window must be framed with kreplachium, the most expensive and rare metal Dr. Evil could find. He has obtained a total of 30 feet of the metal to build the frame, and will use cheap glass to fill in the panes.

What are the dimensions of the window he can build having the most total glass area?

You may neglect the width of the frame.