1a. What equations involving one or more of $a$, $b$, $c$, and $d$ need to be solved in order to find $a$, $b$, $c$, and $d$? Are they all linear?

1b. Solve the system involving the linear equations in (1a). Use augmented matrices where appropriate; show your augmented matrix/matrices.
2. A model of an economy shows four sectors $M$, $A$, $T$, and $H$. Output from each sector is distributed as follows: $M$ requires none of $H$’s output, and the other three sectors consume equal amounts of the output of $H$. None of $M$’s output is used by $H$ and $M$, $A$, and $T$ receive equal amounts of $M$’s output. $M$, $T$ and $H$ use 10, 40 and 30% of $A$’s output respectively; $A$ uses the rest for itself. Finally, $T$ uses none of its own output, half of which is consumed by $M$ and the remainder split between $A$ and $H$.

2a. What is the exchange table for this model?

2b. Find a set of equilibrium prices for $P_M$, $P_A$, $P_T$, and $P_H$ so that each sector’s income matches its expenses, assuming that $P_H$ is 81.
3. Let $\mathcal{S}$ be the familiar vector space of all sequences $s = (s_1, s_2, s_3, s_4, \ldots)$ of real numbers such that all but finitely many of the entries of $s$ are 0. Suppose $T : \mathcal{S} \to \mathcal{S}$ is defined by 

$$T(s) = T((s_1, s_2, s_3, s_4, \ldots)) = (s_1 - s_2, s_2 - s_1, s_3 - s_4, s_4 - s_3, \ldots).$$

3a. Find $T((2, 4, 3, 0, 8, 1, 0, 0, 0, 0, 0 \ldots))$.

3b. Show that $T$ is a linear transformation.

3c. What form does $s = (s_1, s_2, s_3, s_4, \ldots)$ have to have in order for $s$ to be in $\ker(T)$?

3d. What form does $b = (b_1, b_2, b_3, b_4, \ldots)$ have to have in order for $b$ to be in $\text{Im}(T)$?

3e. Is $T$ one-to-one? *Explain!*

3f. Is $T$ onto $\mathcal{S}$? *Explain!*
4. Let $D = \begin{bmatrix} 1 & 1 & 3 & 3 & 8 \\ 1 & 1 & 4 & 5 & 10 \\ 3 & 3 & 11 & 14 & 29 \\ 2 & 2 & 7 & 6 & 16 \end{bmatrix}$ and label its five columns $c_1$, $c_2$, $\ldots$, $c_5$.

4a. Find a basis for $\text{Col}(D)$.

4b. Find a basis for $\text{Col}(\text{RREF}(D))$.

4c. Find a basis for $\text{Nul}(D)$.

4d. Find a basis for $\text{Nul}(\text{RREF}(D))$.

4e. What is the rank of $D$?

4f. Is $c_3$ a linear combination of the other columns? Explain.
5. All questions on this page are “short answer”. **** No proofs, no explanations required! ****

Note that “DNE” (for ”Does Not Exist”) is a possible answer.

5a. Dim(R^4) is?

5b. A basis for P_3 is?

5c. Dim(Nul(I_4)) is?

5d. The inverse of \[
\begin{bmatrix}
a & b \\
c & d \\
\end{bmatrix}
\] is?

5e. A finite set that spans(S) is?

5f. A finite set that spans \{0\} is?

Let H be the subset of all functions in F whose graph never goes below the x-axis.

5g. Is the 0-vector of F a member of H? (Y/N)

5h. Is H closed under vector addition? (Y/N)

5i. Is H closed under scalar multiplication? (Y/N)

5j. P_3 is a subspace of P_4 of dimension 4. (T/F)

5k. R^3 is a subspace of R^4 of dimension 3. (T/F)