Math 206 Quiz Eight

Name: _______________________

1. Given the vector field $\vec{F}(x, y, z) = (y, x, 1)$
   a. Prove that $\vec{F}$ is path independent in $\mathbb{R}^3$ by finding a potential function for $\vec{F}$.
   b. If $C$ is the path in $\mathbb{R}^3$ parametrized by $\vec{g}(t) = (t^4, \sin(\frac{\pi t}{2}), t)$ for $0 \leq t \leq 1$, calculate $\int_C \vec{F} \cdot d\vec{x}$.

2. Prove that if $U \subset \mathbb{R}^n$ is an open connected set and $\vec{F} : U \to \mathbb{R}^n$ is a continuous vector field, then $\vec{F}$ is path independent on $U$ if and only if $\oint_C \vec{F} \cdot d\vec{x} = 0$ for all closed paths $C$ that lie in $U$. 