1. Let $\beta = \{1, 2 - t, t^2 + t^3, t^3\}$ be a basis for $P_3$. Suppose you are given the following coordinate vector:

$$[\bar{p}(t)]_\beta = \begin{bmatrix} 2 \\ 1 \\ 3 \\ -1 \end{bmatrix}$$

What is $\bar{p}(t)$?

$$2 \cdot 1 + 1 \cdot (2 - t) + 3 \cdot (t^2 + t^3) - 1 \cdot t^3 = 4 - t + 3t^2 + 2t^3$$

2. Let $\Delta = \{1 + t, 2t, t + t^2, t^3\}$ be a basis for $P_3$. What is the coordinate vector for the vector $\bar{p}(t) = 1 + 4t + 3t^2$?

$$\bar{p}(t) = 1 + 4t + 3t^2 = 1 \cdot (1 + t) + 0 \cdot 2t + 3 \cdot (t + t^2) + 0 \cdot t^3$$

$$[\bar{p}(t)]_\beta = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

3. Determine whether the vectors $\{1+2t^2, 4+t, t+5t^2\}$ in $P_2$ are linearly independent or linearly dependent by using a coordinate vector argument.

Use the ordered basis $\{1, t, t^2\}$ to determine the coordinate vectors.

$$\begin{bmatrix} 1 & 4 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

There is a pivot in every column, so these vectors are linearly independent.

4. $B = \begin{bmatrix} 3 & 1 & 2 & 3 \\ 0 & -2 & 4 & 8 \\ 12 & 18 & 8 & 4 \end{bmatrix}$

$$\text{rref}(B) = \begin{bmatrix} 1 & 0 & 0 & 1/21 \\ 0 & 1 & 0 & -4/7 \\ 0 & 0 & 1 & 12/7 \end{bmatrix}$$

(a) What is the dimension of the Col($B$)? How do you know?

Three dimensional. There are three pivot columns.

(b) What is the dimension of the Nul($B$)? How do you know?

One dimensional. There is one free variable.

(c) The Nul($B$) is a subspace of $\mathbb{R}^k$. What is $k$? $k = 4$

(d) The Col($B$) is a subspace of $\mathbb{R}^p$. What is $p$? $p = 3$