Here are some facts:

Let \( A = \begin{bmatrix}
  -4 & -8 & -16 & 8 & -10 & -14 \\
  6 & 19 & 31 & 16 & 26 & 20 \\
  5 & 16 & 26 & 14 & 20 & 7 \\
  3 & 10 & 16 & 10 & 17 & 26 \\
\end{bmatrix} \), and let

\[
\begin{bmatrix}
  b_1 \\
  b_2 \\
  b_3 \\
  b_4 \\
\end{bmatrix}, \quad 
\begin{bmatrix}
  30 \\
  37 \\
  0 \\
  79 \\
\end{bmatrix}, \quad 
\begin{bmatrix}
  3 \\
  4 \\
  -7 \\
  3 \\
\end{bmatrix}, \quad \begin{bmatrix}
  -13 \\
  24 \\
  4 \\
  3 \\
\end{bmatrix}
\]

Let the columns of \( A \) be \( c_1 \ldots c_6 \), and let the columns of \( \text{rref}(A) \) be \( k_1 \ldots k_6 \). Let \( R = \text{rref}(A) \).

**Fact 1.** The \( \text{rref} \) of \( [A \mid I_4] \) is

\[
\begin{bmatrix}
  1 & 0 & 2 & -10 & 0 & 7 & 0 & 8 & -7 & -4 \\
  0 & 1 & 1 & 4 & 0 & -8 & 0 & -25/9 & 8/3 & 10/9 \\
  0 & 0 & 0 & 0 & 1 & 5 & 0 & 2/9 & -1/3 & 1/9 \\
  0 & 0 & 0 & 0 & 0 & 1 & 12 & -10 & -6 \\
\end{bmatrix}
\]

**Fact 2.** The matrix product

\[
\begin{bmatrix}
  -4 & -8 & -16 & 8 & -10 & -14 \\
  6 & 19 & 31 & 16 & 26 & 20 \\
  5 & 16 & 26 & 14 & 20 & 7 \\
  3 & 10 & 16 & 10 & 17 & 26 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
  30 \\
  37 \\
  0 \\
  79 \\
\end{bmatrix}
\]

is

\[
\begin{bmatrix}
  102 & -10 & 0 & 7 & 0 & 8 & -7 & -4 \\
  24 & 4 & -7 & 0 & 37 \\
  3 & 3 & 0 & 0 & 79 \\
  -25 & 2 & 5 & 3 \\
\end{bmatrix}
\]

**Fact 3.** Finally, the \( \text{rref} \) of

\[
\begin{bmatrix}
  -4 & -8 & -16 & 8 & -10 & -14 & 30 \\
  6 & 19 & 31 & 16 & 26 & 20 & 37 \\
  5 & 16 & 26 & 14 & 20 & 7 & 0 \\
  3 & 10 & 16 & 10 & 17 & 26 & 79 \\
\end{bmatrix}
\]

is

\[
\begin{bmatrix}
  1 & 0 & 2 & -10 & 0 & 7 & -20 \\
  0 & 1 & 1 & 4 & 0 & -8 & -15 \\
  0 & 0 & 0 & 0 & 1 & 5 & 17 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

The first question is here:

1. What does it mean (ie, what’s the definition) for a set \( B \) of vectors \( \{v_1 \ldots v_p\} \) to be a basis of a subspace \( H \) of a vector space.

   \[ \text{The set } B \text{ must...} \]
   \[ (1) \text{ span } H \]
   \[ (2) \text{ be a L.I. set} \]
2. Let \( A, b, u, v, \) and \( w \) be as on page one of this Quiz.

(2A) What conditions, if any, do the entries \( b_1, \ldots, b_4 \) of \( b \) have to satisfy in order for \( b \) to be in \( \text{Col}(A) \)?

From fact 1, we see
\[
0 = b_1 + 12b_2 - 10b_3 - 6b_4
\]

(2B) Verify that \( u \) is in \( \text{Col}(A) \) according to the conditions in (2A).

\[
\begin{bmatrix}
20 \\
37 \\
0 \\
29
\end{bmatrix}
\]

\( \text{does } 0 = 30 + 12 \cdot 37 - 10 \cdot 0 - 6 \cdot 29 \) ?

\( = 30 + 444 - 474 = 434 - 474 = 0 \checkmark \)

(2C) Is \( k_1 \) in \( \text{Col}(A) \)? Explain.

\( k_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \) is not in \( \text{Col}(A) \) bec it does not satisfy the conditions found in (2A)

(2D) Are any column vectors of \( A \) in the column space of \( R \)? Explain.

\( \text{NO} \) The only \( \text{O}'s \) in \( R \) tell us that any \( \text{LC} \) of the column vectors of \( R \)
will have the form \( \begin{bmatrix} k \\ k \\ k \\ k \end{bmatrix} \), yet none of the column vectors of \( A \) have this form; their
fourth entries are non-zero.

(2E) Which column vectors of \( A \) form a basis of \( \text{Col}(A) \)? (write your answer in terms of the \( c_i \)'s)

\( \vec{c}_1, \vec{c}_2, \vec{c}_3 \)

(2F) Express \( u \) as a linear combination (LC) of the basis vectors in (2E).

\( \text{write your answer in terms of the } c_i \text{'s, eg } 3c_2 + 4c_5 \)"

\( \text{Fact 3 shows that } -20\vec{c}_1 - 15\vec{c}_2 + 7\vec{c}_5 = \hat{u} \)

(2G) What LC is \( 3c_1 + 4c_2 - 7c_3 + 3c_4 + 2c_5 + 3c_6 \) and how did you find it? (eg, "I used fact 7" or "I used my calculator
to compute..." or "I computed [give the expression] by hand", etc)

\( \text{Fact 2 shows } A \begin{bmatrix} 3 \\ 4 \\ -7 \\ 37 \\ 0 \\ 29 \end{bmatrix} = \begin{bmatrix} 30 \\ 37 \\ 0 \\ 0 \\ 9 \end{bmatrix} = \hat{u}. \)

(2H) What LC is \( -13c_1 + 24c_2 + 4c_3 + 3c_4 - 25c_5 + 5c_6 \) and how did you find it?

\( \text{Fact 2 shows } A \begin{bmatrix} -11 \\ -24 \\ -7 \\ -24 \\ -4 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \)

(2I) Does the result of (2H) say that \( w \) is or is not in \( \text{Nul}(A) \)? Since \( 2H \) shows \( A\hat{w} = 0 \), we have \( \hat{w} \in \text{Nul}(A) \).

(2J) Find a basis for \( \text{Nul}(A) \). The row reduction in fact 1 is useful here:

\( x_1 = -2x_2 + 10x_4 - 7x_6 \)
\( x_2 = -x_3 + 4x_4 + 8x_6 \)
\( x_3 \text{ is free} \)
\( x_4 \text{ is free} \)
\( x_5 = -5x_6 \)
\( x_6 \text{ is free} \)

So \( \text{Nul}(A) \) is all \( \text{LC's of} \begin{bmatrix} -2 \\ -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \), that is, then
3 vectors form
a basis \( \text{Nul}(A) \).

(2K) Express \( w \) as a LC of the basis vectors from (2J)

See the 1st page (not enough room here)