1. Let \( f(x) = \ln(1 - x) \).

(a) The 5th-order Taylor polynomial of \( f(x) \) centered at \( x = 0 \) may be found as follows:

\[
\begin{align*}
f(x) &= \ln(1 - x) \\
f'(x) &= \frac{-1}{1 - x} = -(1 - x)^{-1} \\
f''(x) &= -(1 - x)^{-2} \\
f'''(x) &= -2(1 - x)^{-3} \\
f^{(4)}(x) &= -(3!)(1 - x)^{-4} \\
f^{(5)}(x) &= -(4!)(1 - x)^{-5}
\end{align*}
\]

Therefore, the 5th-order Taylor polynomial of \( f(x) \) centered at \( x = 0 \) is

\[
P_5(x) = f(0) + f'(0)(x - 0) + \frac{f''(0)}{2!}(x - 0)^2 + \frac{f'''(0)}{3!}(x - 0)^3 + \frac{f^{(4)}(0)}{4!}(x - 0)^4 + \frac{f^{(5)}(0)}{5!}(x - 0)^5
\]

\[
= -x - \frac{1}{2}x^2 - \frac{2}{3!}x^3 - \frac{3!}{4!}x^4 - \frac{4!}{5!}x^5
\]

Therefore, \( P_5(x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} \)

(b) Use the Taylor polynomial \( P_5(x) \) to find an estimate for \( \ln(0.5) \).

First notice that \( f(0.5) = \ln(1 - 0.5) = \ln(0.5) \).

Therefore, to estimate \( \ln(0.5) \) we need to evaluate \( P_5(x) \) when \( x = 0.5 \).

\[
P_5(0.5) = -0.5 - \frac{(0.5)^2}{2} - \frac{(0.5)^3}{3} - \frac{(0.5)^4}{4} - \frac{(0.5)^5}{5} \approx -0.689.
\]

In other words, \( \ln(0.5) \approx -0.689 \).

2. To determine whether \( \int_0^{\pi/2} \frac{\sin x}{\sqrt{\cos x}} \, dx \) converges or diverges we first need to recognize that

\[ f(x) = \frac{\sin x}{\sqrt{\cos x}} \] is undefined at \( x = \frac{\pi}{2} \). Therefore, the integral is improper and must be evaluated as shown below.

\[
\int_0^{\pi/2} \frac{\sin x}{\sqrt{\cos x}} \, dx = \lim_{b \to \pi/2} \int_0^b \frac{\sin x}{\sqrt{\cos x}} \, dx
\]

\[
= \lim_{b \to \pi/2} \int_1^{\cos b} \frac{-du}{\sqrt{u}} \quad \text{(where } u = \cos x \text{ and } du = -\sin x \, dx)\]

\[
= \lim_{b \to \pi/2} \left[ -2\sqrt{u} \right]_1^{\cos b} = \lim_{b \to \pi/2} \left[ -2\sqrt{\cos b} + 2\sqrt{1} \right] = 2 \quad \text{(since } \sqrt{\cos b} \to 0 \text{ as } b \to \frac{\pi}{2})
\]

Therefore, the improper integral converges, in fact,

\[
\int_0^{\pi/2} \frac{\sin x}{\sqrt{\cos x}} \, dx = 2
\]