Math 206A: Fall 2011
Quiz 3: October 26

For full credit you must show your work. Good Luck!

1. Show that \( \lim_{(x,y) \to (0,0)} \frac{x^2}{x^2 + y^2} \) does not exist.

\[
\lim_{(x,y) \to (0,0)} \frac{x^2}{x^2 + y^2} = \lim_{y \to 0} \frac{x^2}{x^2} = 1 = 1
\]

Since \( 1 \neq \frac{1}{2} \) we see that the function has different limits as \((x,y) \to (0,0)\) on different paths.

Thus limit does not exist.

2. Show that \( \lim_{(x,y) \to (0,0)} \frac{x^2y^2}{x^2 + y^2} \) exists and find the limit.

\[
0 \leq \frac{x^2y^2}{x^2 + y^2} = x^2 \frac{y^2}{x^2 + y^2} \leq x^2 \frac{x^2 + y^2}{x^2 + y^2} = x^2
\]

So by squeeze theorem

\[
0 = \lim_{(x,y) \to (0,0)} 0 \leq \lim_{(x,y) \to (0,0)} \frac{x^2y^2}{x^2 + y^2} \leq \lim_{(x,y) \to (0,0)} x^2 = 0
\]

\( \Rightarrow \) Limit exists and is 0.
3. Let \( f(x, y) = x^2 y + y^3 \).

Find \( \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}, \frac{\partial^2 f}{\partial x \partial y} \).

\[
\frac{\partial f}{\partial x} = 2xy, \quad \frac{\partial f}{\partial y} = x^2 + 3y^2
\]

\[
\frac{\partial^2 f}{\partial x^2} = 2y, \quad \frac{\partial^2 f}{\partial y^2} = 6y
\]

\[
\frac{\partial^2 f}{\partial x \partial y} = 2x
\]

4. Let \( f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}} \). Find \( \frac{\partial f}{\partial x} \).

\( f(x, y) = xy (x^2 + y^2)^{-1/2} \)

\[
\frac{\partial f}{\partial x} = y \left( x^2 + y^2 \right)^{-1/2} + (xy) \left( -\frac{1}{2} \right) (x^2 + y^2)^{-3/2} \left( 2x \right)
\]

\[
= \frac{y}{\sqrt{x^2 + y^2}} - \frac{x^2 y}{(\sqrt{x^2 + y^2})^3} = \frac{y (x^2 + y^2) - x^2 y}{(\sqrt{x^2 + y^2})^3} = \frac{y^3}{(\sqrt{x^2 + y^2})^3}
\]