1. Find the derivative of each of the following functions. You do not need to simplify your answers.

1A. \( p(x) = \frac{x^6 + \cos(x)}{\sin(x) + x^5} \)

By the quotient rule, \( p'(x) = \frac{(\sin(x) + x^5) \cdot (x^6 + \cos(x))' - (x^6 + \cos(x)) \cdot (\sin(x) + x^5)'}{(\sin(x) + x^5)^2} \)

\[ = \frac{(\sin(x) + x^5)(6x^5 - \sin(x)) - (\cos(x) + 5x^4)(x^6 + \cos(x))}{(\sin(x) + x^5)^2} \]

1B. \( m(x) = \sin(e^{2x}) \)

by the chain rule,

\[ m'(x) = \cos(e^{2x}) \cdot (e^{2x})' \]

but this part also requires the chain rule since \( (2x)^2 \) is the "inside function"; we get \( (e^{2x})' = e^{2x} \cdot (2x)' = e^{2x} \cdot 2 \)

finally: \( m'(x) = \cos(e^{2x}) \cdot e^{2x} \cdot 2 \)

Alternate Notation:

\[ m(x) = \sin \left( \exp(2x) \right) \]

\[ so \\ m'(x) = \cos \left( \exp(2x) \right) \cdot \exp(2x)(2x)' \]

\[ = \cos(2x) \cdot \exp(2x)(2x)' \]

\[ = \cos(2x) \cdot \exp(2x) \cdot 2 \]

\[ = \cos e^{2x} \cdot e^{2x} \cdot 2 \]

1C. \( k(x) = \ln \left( x^2 + \sqrt{x^2 + \cos(7x)} \right) \)

\[ k'(x) = \frac{1}{x^2 + \left( x^2 + \cos(7x) \right)^{1/2}} \cdot \left[ \left( x^2 + \left( x^2 + \cos(7x) \left( \frac{1}{2} \right) \right) \right)^{-1/2} \cdot \left( 2x + \frac{1}{2} \left( x^2 + \cos(7x) \right)^{-1/2} \cdot \left( 2x - \sin(7x) \cdot 7 \right) \right) \right] \]

1D. \( g(x) = x^4 (x^5 + \log_2 x) \)

\[ g'(x) = (4x^3)(x^5 + \log_2 x) + (x^4)(5x^4 + \frac{1}{\ln 2} \cdot \frac{1}{x}) \]
2. The equation \( x^3 + y^4 = x^2 y^2 + 1 \) implicitly defines \( y \) as a function of \( x \), and a graph of this equation is shown at the bottom of the page.

2A. Use implicit differentiation to find \( y' \).

\[
3x^2 + 4yy'y' = 2xy^2 + x^2y' + 0
\]

\[
\frac{4y^3y' - x^2y}{y' (4y^3 - x^2y)} = 2xy^2 - 3x^2
\]

\[
y' = \frac{2xy^2 - 3x^2}{4y^3 - x^2y}
\]

2B. The graph implies that \((1, 1)\) is a solution of the equation; show that \((1, 1)\) does indeed satisfy the equation.

\[
does \quad 1^3 + 1^4 = 1^2 \cdot 1^2 + 1 \quad ? \quad \text{does} \quad 1 + 1 = 1 \cdot 1 + 1 \quad ? \quad \text{does} \quad 2 = 1 + 1 \quad ? \quad \text{yes.}
\]

2C. Use the answer to 2A to find the slope of the graph of the equation at \((1, 1)\).

\[
\left. \frac{dy}{dx} \right|_{(1,1)} = \frac{2 \cdot 1 \cdot 1^2 - 3 \cdot 1^2}{4 \cdot 1^2 - 1 \cdot 2 \cdot 1} = \frac{2 - 3}{4 - 2} = \frac{-1}{2}
\]

(This seems reasonable if you sketch the tangent line here, as I've done.)