1. Find the Taylor polynomial \( P_2(x) \) of degree 2 for \( f(x) = \sqrt{x} \) in powers of \((x - 8)\). Organize your work in finding \( c_0, c_1, \) etc., into a table like the kind we've made in class. Leave your \( c_i \)'s as fractions; don't use their decimal equivalents as you assemble your polynomial.

<table>
<thead>
<tr>
<th>K</th>
<th>( f^{(k)}(8) )</th>
<th>( f^{(k)}(x) )</th>
<th>( c_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( x^{\frac{1}{2}} )</td>
<td>( 8^{\frac{1}{2}} = \sqrt{8} = 2 )</td>
<td>( c_0 = \frac{2}{0!} = 2 )</td>
</tr>
<tr>
<td>1</td>
<td>( \frac{1}{3} x^{-\frac{1}{3}} )</td>
<td>( \frac{1}{3} \cdot 8^{-\frac{1}{3}} = \frac{1}{3} \cdot \frac{2}{3} = \frac{1}{3} \cdot \frac{1}{2} )</td>
<td>( c_1 = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6} )</td>
</tr>
<tr>
<td>2</td>
<td>( -\frac{4}{9} x^{-\frac{7}{3}} )</td>
<td>( -\frac{4}{9} \cdot 8^{-\frac{7}{3}} = -\frac{2}{9} \cdot \frac{1}{8} \cdot \frac{1}{2} = -\frac{1}{9} \cdot \frac{1}{2} = -\frac{1}{18} )</td>
<td>( c_2 = \frac{1}{18} \cdot \frac{1}{2} = -\frac{1}{36} )</td>
</tr>
</tbody>
</table>

So \( P_2(x) = 2 + \frac{1}{12} (x-8) - \frac{1}{288} (x-8)^2 \).

2. What's the "error guarantee" for \( P_2 \) as used to approximate \( f \) on the interval \([5, 10]\)? Show all your work. Give your \( K_3 \) to four places after the decimal point. (Hint: Plot the appropriate derivative in the window \([5, 10] \times [-0.01, 0.01]\) to get started.)

The error guarantee is \( \frac{K_3 |x-8|^3}{3!} \) where:

1) \( K_3 \) is chosen to satisfy \( |f^{(3)}(x)| \leq K_3 \) for all \( x \) in \([5, 10]\).

Now, \( f^{(3)}(x) = \frac{10}{27} x^{-10/3} \); is "maxed" at 5; see the plot.

At \( x=5 \), the value of \( f^{(3)}(5) = 0.0050665954... \); so to FOUR places after the decimal point, we should choose \( K_3 = 0.0051 \).

2) \( |x-8|^3 \) needs to be chosen to have the max. value on \([5, 10]\):

(i.e. what's the biggest distance any point in \([5, 10]\) from 8)

We take this to be 3, so the error guarantee becomes:

\[
\frac{0.0051 \times 3^2}{3!} = 0.02295
\]

3. Find both \( P_2(5) \) and \( f(5) \) on your calculator; write them down to as many places as your calculator shows. Label which is which.

\[
P_2(5) = 2 + \frac{1}{12} (5-8) - \frac{1}{288} (5-8)^2 = 1.71875
\]

and \( f(5) = \sqrt{5} = 1.709975947... \)

4. What's the difference between the two numbers in question 3? Is it indeed less than the "error guarantee" in question 2?

\[
|P_2(5) - f(5)| = 0.008779... \text{ and this is less than } 0.02295.
\]