1. Suppose $T(w, x, y, z) = x^3 z + 2xy - 4wy^2 + 8w\sqrt{z} + \sin(wx)$.

   (a) What is the “direction” of the maximum rate of change of $T$ when $w = 0, x = -1, y = 2, z = 1$?
   (Hint: your answer will be a 4-dimensional vector like $(a, b, c, d)$.)

   **Direction of max rate of change is $\nabla T$.**

   $T_w = -4y^2 + 8\sqrt{z} + x\cos(wx)$
   $T_w(0, -1, 2, 1) = -16 + 8 - 1 = -9$
   $T_x = 3x^2 z + 2y + w\cos(wx)$
   $T_x(0, -1, 2, 1) = 3 + 4 + 0 = 7$
   $T_y = 2x - 8wy$
   $T_y(0, -1, 2, 1) = -2$
   $T_z = x^3 + \frac{8}{2} w\frac{\sqrt{z}}{z}$
   $T_z(0, -1, 2, 1) = -1 + 0 = -1$

   $\nabla T = (-9, 7, -2, -1)$

   (b) What is the maximum rate of change of $T$ when $w = 0, x = -1, y = 2, z = 1$?

   **Maximum rate of change is $||\nabla T||$.**

   $||\nabla T|| = \sqrt{(-9)^2 + (7)^2 + (-2)^2 + (-1)^2} = \sqrt{135}$
2. True or false? (Carefully justify your answer.) If \( \mathbf{\hat{a}} \) is a unit vector and the level curves of \( f(x,y) \) are given below, then at point \( P \) we have \( f_\mathbf{\hat{a}}(P) = \| \text{grad } f \| \cos \theta \).

\[ \mathbf{\hat{a}} \text{ appears to be } \perp \text{ to the level set so grad } f(P) \text{ points in the same direction as } \mathbf{\hat{a}}. \]

By definition
\[ f_\mathbf{\hat{a}}(P) = \text{grad } f(P) \cdot \mathbf{\hat{a}} \]

Now using property of dot product
\[ f_\mathbf{\hat{a}}(P) = \| \text{grad } f(P) \| \| \mathbf{\hat{a}} \| \cos \theta \]
\[ = \| \text{grad } f(P) \| \cos \theta \]
\[ \because \| \mathbf{\hat{a}} \| = 1. \]

30 \quad \boxed{\text{TRUE}}

3. Find the equation of the plane tangent to the surface \( 3xyz = 15 - x^3 - y^3 - z^3 \) at \( (1, -3, 2) \).

This surface is a level set of
\[ T(x, y, z) = 3xyz + x^3 + y^3 + z^3. \]

(Or if \( T(x, y, z) = 3xyz + x^3 + y^3 + z^3 \).

\[ \nabla T(1, -3, 2) \text{ is } \perp \text{ to the surface at } (1, -3, 2) \]

\[ T_x = 3yz + 3x^2 \quad \rightarrow \quad T_x(1, -3, 2) = -18 + 3 = -15 \]
\[ T_y = 3xz + 3y^2 \quad \rightarrow \quad T_y(1, -3, 2) = 6 + 27 = 33 \]
\[ T_z = 3xy + 3z^2 \quad \rightarrow \quad T_z(1, -3, 2) = -9 + 12 = 3 \]

\[ \nabla T(1, -3, 2) = -15\mathbf{i} + 33\mathbf{j} + 3\mathbf{k} \]

A plane tangent to the surface is \( \perp \) to \( \nabla T(1, -3, 2) \),

the eqn of such a plane is

\[ 0 = -15(x - 1) + 33(y + 3) + 3(z - 2) \]

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**Note:** There is an error in this problem. \( (1, -3, 2) \) is not even on the surface. \( 3(1)(-3)(2) = -18 \) but \( 15 - x^3 - y^3 - z^3 = 33 \). The solution written above shows the technique that you would use if the given point was on the surface.