Problem 1. (16 points) Complete only four of the following five problems. Circle all of the listed techniques you use.

(a) (4 points) \( f(x) = \sin(\sqrt{x}) - \frac{1}{x} \). What is \( f'(x) \)?

\[
f'(x) = \cos(\sqrt{x}) \frac{1}{2\sqrt{x}} + \frac{1}{x^2}.
\]

(b) (4 points) \( h(t) = \sqrt{1 - \sin^4(t)} \). What is \( h'(t) \)?

\[
h'(t) = \frac{1}{2\sqrt{1 - \sin^4(t)}}(-4\sin^3(t)\cos(t)).
\]

(c) (4 points) \( p(x) = \frac{2x}{e^x - e^{-x}} \). What is \( p'(x) \)?

\[
p'(x) = \frac{2(e^x - e^{-x}) - 2x(e^x + e^{-x})}{(e^x - e^{-x})^2}.
\]

(d) (4 points) \( g(x) = x^{17}e^{-x} \). What is \( \frac{dg}{dx} \)?

\[
\frac{dg}{dx} = 17x^{16}e^{-x} - x^{17}e^{-x}.
\]

(e) (4 points) \( a^{12} + ab^2 - 10 = 0 \). What is \( \frac{da}{db} \)?

Take \( \frac{d}{db} \) (the derivative with respect to \( b \)) on both sides:

\[
12a^{11} \frac{da}{db} + \frac{da}{db}b^2 + 2ab = 0
\]

\[
\frac{da}{db}(12a^{11} + b^2) = -2ab
\]

\[
\frac{da}{db} = \frac{-2ab}{12a^{11} + b^2}.
\]
Problem 2. (8 points) You have a limited amount of land on which to grow \( w \) acres of watermelon and \( y \) acres of yams, constrained by the equation
\[
w^2 + y^3 - y - 15 = 0.
\]
If you’re currently growing \( w = 3 \) acres of watermelons and \( y = 2 \) acres of yams, compute \( \frac{dy}{dw} \) and explain what it means in terms of watermelons and yams, including units.

Use implicit differentiation: take the derivative of both sides of the equation with respect to \( w \).
\[
2w + 3y^2 \frac{dy}{dw} - \frac{dy}{dw} = 0
\]
\[
\frac{dy}{dw}(3y^2 - 1) = -2w
\]
\[
\frac{dy}{dw} = \frac{-2w}{3y^2 - 1}.
\]
If \( w = 3 \) and \( y = 2 \), then
\[
\frac{dy}{dw} = \frac{-2(3)}{3(2)^2 - 1} = \frac{-6}{11} \approx -0.545 \text{ acres yams per acre watermelons}.
\]
That is, for each additional acre of watermelons we wish to grow, we must sacrifice 0.545 acres of yams.

Problem 3. (6 points) Two students are asked to compute the derivative of the function \( L(x) = \frac{x^2}{\sin^3 x} \). Both of them make mistakes. In the space provided, briefly explain what each student has done wrong. (Hint: their algebra is fine.)

(a) (3 points)
\[
L'(x) = \frac{2x}{3 \sin^2 x \cos x} = \frac{2}{3} \frac{x}{\sin^2 x \cos x}.
\]
A classic “rookie mistake;” instead of using the quotient rule, this student has differentiated the numerator, then differentiated the denominator.
(This is easy to confuse after you learn l’Hôpital’s rule, which uses that technique to solve a very different problem.)

(b) (3 points)
\[
L'(x) = \frac{x^2 (3 \sin^2 x \cos x) - 2x \sin^3 x}{(3 \sin^2 x \cos x)^2} = \frac{x^2}{3 \sin^2 x \cos x} - \frac{2x \sin^3 x}{(3 \sin^2 x \cos x)^2}.
\]
This student attempted the quotient rule, but made two major mistakes:
Derivatives don’t belong downstairs! The denominator should be \((\sin^3 x)^3\).
The order of terms in the numerator is backwards.

P.S. The correct derivative is
\[
L'(x) = \frac{2x \sin^3 x - 3x^2 \sin^2 x \cos x}{\sin^6 x}.
\]