MATH 105A,C - CALCULUS I
FALL 2010

QUIZ 4

NAME:

Show ALL your work CAREFULLY.

1. Let \( f(x) = 2 \ln x + 3 \cos x \). Find \( f'(x) \).

Taking the derivative of \( f \), we have

\[
f'(x) = 2 \cdot \frac{1}{x} + 3 \cdot (-\sin x).
\]

It follows that \( f'\left(\frac{\pi}{2}\right) = 2 \cdot \frac{2}{\pi} - 3 \sin(\pi/2) = \frac{4}{\pi} - 3 \).

2. Find the exact value of the following limit.

\[
\lim_{x \to 1} \frac{3^x - 3}{x - 1}
\]

Let \( g(x) = 3^x \). Then the limit

\[
\lim_{x \to 1} \frac{3^x - 3}{x - 1} = \lim_{x \to 1} \frac{g(x) - g(1)}{x - 1} = g'(1).
\]

Since \( g'(x) = (\ln 3) \cdot 3^x \), we have \( g'(1) = 3 \ln 3 \).

3. Find an equation of the line tangent to the graph of \( \log_2 x \) at \( x = 4 \).

First,

\[
\frac{d}{dx} \log_2 x = \frac{1}{\ln 2} \cdot \frac{1}{x}.
\]

The slope of the tangent line is \( \frac{1}{\ln 2} \cdot \frac{1}{4} = \frac{1}{4 \ln 2} \). Since the tangent line goes through the point \( (4, \log_2 4) = (4, \log_2 2^2) = (4, 2) \), the tangent line has an equation

\[
(y - 2) = \frac{1}{4 \ln 2} \cdot (x - 4) \quad \text{or} \quad y = \frac{1}{4 \ln 2} x - \frac{1}{\ln 2} + 2.
\]

Date: October 15, 2010.