1. Suppose \( M = \begin{bmatrix} 5 & 3 & 1 \\ 20 & 10 & b \end{bmatrix}, \quad N = \begin{bmatrix} c & s & 2 \\ c & t & 5 \end{bmatrix} \) and \( MN = \begin{bmatrix} 36 & 36 & a \\ z & 280 & 240 \end{bmatrix} \).

Find the values of all these unknowns. Hint: it will be easy if you find them in more-or-less alphabetical order. There are many equations you can write down, but aim for those with as few unknowns as possible.

\[
\begin{align*}
A &= 5 \cdot 2 + 3 \cdot 5 + 1 \cdot 3 = 10 + 15 + 3 = 28 \\
20 \cdot 2 + 10 \cdot 5 + 6 \cdot 3 &= 240 \\
40 + 50 + 3b &= 240 \\
3b &= 240 - 90 = 150 \\
b &= 50 \\
5c + 3c + 1c &= 36 \\
9c &= 36 \\
c &= 4
\end{align*}
\]

Answers: \(a = 28\) \(b = 50\) \(c = 4\)

\[
\begin{align*}
\begin{aligned}
\begin{cases}
5s + 3t + 3 &= 36 \\
20s + 10t + 3b &= 280 \\
20s + 10t &= 130
\end{cases}
\rightarrow
\begin{bmatrix} 5 & 3 & 33 \\ 2 & 1 & 13 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow s &= 6 \\
t &= 1
\end{aligned}
\end{align*}
\]

\[
\begin{align*}
z &= "2 nd \ row \times 1 st \ col" = 20c + 10c + bc = (20 + 10 + 50)(4) \\
s &= 6 \\
t &= 1 \\
z &= 320
\end{align*}
\]

BONUS: How many pairs of numbers have to multiplied to find the product of a \(3 \times 7\) by a \(7 \times 10\) matrix? Explain. (No credit for a guess).

The product has \(3 \times 10 = 30\) entries, each of which requires \(7\) pairs of numbers to be multiplied: \(210\) total multiplications.