MATH 205A,B LINEAR ALGEBRA - PROF. P. WONG

EXAM I - OCTOBER 7, 2015

NAME: Section: (Circle one) A(8:00) B(9:30)

Instruction: Read each question carefully. Explain ALL your work and give reasons to support your answers.

Advice: DON’T spend too much time on a single problem.

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<thead>
<tr>
<th>Problems</th>
<th>Maximum Score</th>
<th>Your Score</th>
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<tbody>
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1. Consider the following system of linear equations

\[
\begin{align*}
4x_1 - 2x_2 + 7x_3 &= -5 \\
8x_1 - 3x_2 + 10x_3 &= -3.
\end{align*}
\]

(a) Find the solutions to the system (1), if it is consistent.

(b) Find the solutions to the homogeneous system

\[
\begin{align*}
4x_1 - 2x_2 + 7x_3 &= 0 \\
8x_1 - 3x_2 + 10x_3 &= 0.
\end{align*}
\]
2. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by
   \[ T(x_1, x_2) = (3x_1 + 2x_2, -x_1 + 3x_2, x_1 + x_2). \]

(a) Find all $\vec{x}$ such that $T(\vec{x}) = \vec{0}$.

(b) Determine whether $T$ is one-to-one. Justify your answer.

(c) Determine whether $T$ is onto. Justify your answer.
3. Let

\[ B = \begin{bmatrix} 1 & -2 & -1 \\ -2 & 5 & 3 \end{bmatrix}. \]

(a) Are the columns of \( B \) linearly independent? Justify your answer.

(b) Do the columns of \( B \) span \( \mathbb{R}^2 \)? Justify your answer.

(c) Write a formula for the linear transformation \( T : \mathbb{R}^3 \to \mathbb{R}^2 \) so that \( T(\vec{x}) = B\vec{x} \) for any vector \( \vec{x} \) in \( \mathbb{R}^3 \).
4. Use elementary row operations to find the inverse $A^{-1}$ of the following invertible matrix

$$A = \begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 5 \end{bmatrix}.$$ 

(Show all your steps.)
5. (a) Let \( S : \mathbb{R}^2 \to \mathbb{R}^2 \) be a transformation given by

\[
S(x, y) = (1 - xy, x + y).
\]

Determine whether \( S \) is a \textbf{linear} transformation. Explain.

(b) Let \( A = \begin{bmatrix} 1 & -3 & 0 \\ -4 & 1 & 1 \end{bmatrix} \) and \( B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ -1 & 1 \end{bmatrix} \). Find (i) \( AB \); (ii) \( BA^T \); and (iii) \( A - B^T \).