Mathematics 105  
Exam I  
October 7, 2011

<table>
<thead>
<tr>
<th>Problem</th>
<th>Possible</th>
<th>Actual</th>
</tr>
</thead>
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<td>1</td>
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<tr>
<td>5</td>
<td>10</td>
<td></td>
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<tr>
<td>Total</td>
<td>100</td>
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You must show all work to receive credit.  
No electronic devices other than calculators are permitted.  
Give exact answers (such as $\ln 5$ or $e^2$) unless requested otherwise.
1. Consider the following tabular data of some function.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>1.043</td>
<td>1.312</td>
<td>1.629</td>
<td>2.000</td>
<td>2.431</td>
<td>2.928</td>
<td>3.497</td>
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</table>

(a) Provide an estimate of $f'(1)$ using the forward difference approximation with $h = 0.1$.

(b) Provide an estimate of $f'(1)$ using the centered difference approximation with $h = 0.1$.

(c) A more accurate numerical estimation of the derivative is given by the following rule:

$$f'(a) \approx \frac{-f(a + 2h) + 8f(a + h) - 8f(a - h) + f(a - 2h)}{12h}.$$

Use this rule to estimate $f'(1)$ with $h = 0.1$. 
2. Consider the function \( f(x) = x^3 + x \).

(a) Find \( f'(x) \) using the limit definition of the derivative. Hint: \((a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\).

(b) State the power rule for derivatives.

(c) Does the answer you give in part (a) match the answer provided by the use of the power rule? If not, now show that you can use the power rule to find \( f'(x) \).

(d) Evaluate \( f'(1) \).

(e) How does this compare to the numerical estimates found in problem 1? This function is the function used to generate the tabular data.
3. Consider the function:

\[ g(x) = \begin{cases} 
  x^2 - 1 & \text{if } x \geq 0 \\
  1 - x^2 & \text{if } x < 0 
\end{cases} \]

(a) What is \( g'(x) \)?

(b) Evaluate \( \lim_{x \to 0^+} g(x) \).

(c) Evaluate \( \lim_{x \to 0^-} g(x) \).

(d) Is \( g(x) \) continuous at \( x = 0 \)?

(e) Evaluate \( \lim_{x \to 0^+} g'(x) \).

(f) Evaluate \( \lim_{x \to 0^-} g'(x) \).

(g) In light of the above, check your answer to part (a). Does \( g'(0) \) exist? Why or why not?
4. Correct the following sentences by changing the latter half of the sentence.

(a) A function is increasing so the second derivative must be positive.

(b) The derivative is negative so the second derivative must be positive.

(c) The derivative crosses the x-axis means the second derivative is changing concavity.

(d) The derivative is positive so the function must lie above the x-axis.

(e) The second derivative crosses the x-axis so the function must cross the x-axis.

5. Solve the initial value problem given by \( y'' = 6x - 2 \), \( y'(2) = 8 \), and \( y(0) = 4 \).