YOUR GRADE IS BASED ON THE PROCESS AS WELL AS THE FINAL RESULT. SHOW ALL YOUR STEPS CLEARLY SO YOU WILL BE ELIGIBLE FOR THE MOST PARTIAL CREDIT. YOU MAY USE A CALCULATOR, BUT NO NOTES, BOOKS, OR OTHER STUDENTS. GOOD LUCK!

1.) (10 pts.) Given the function

\[ f(x) = \begin{cases} 
-1 & \text{if } x < -2 \\
-x^2 + 3 & \text{if } -2 \leq x \leq 1 \\
2x + 1 & \text{if } 1 < x < 3 
\end{cases} \]

a.) (3 pts.) What is the natural domain of \( f \)?

b.) (2 pts.) What is the range of \( f \)?

c.) (5 pts.) Sketch a graph of \( f \).
2.) (15 pts.) This question will be about functions and graphs.

a.) (5 pts.) Give an example of an exponential function. (This should be a formula.)

b.) (5 pts.) Shift your function from part (a) up by 7 units. What is the resulting function? (Give its formula.)

c.) (5 pts.) Stretch your original function from part (a) horizontally by a factor of 3. What is the resulting function? (Give its formula.)
3.) (15 pts.) Suppose that $D(t)$ represents the depth (in meters) of the water in the deep end of a swimming pool $t$ hours after noon on the hottest day of the summer.

a.) (5 pts.) Interpret the statement $D(1) = 3$.

b.) (5 pts.) Interpret the statement $D'(1) = -0.5$.

c.) (5 pts.) In words, describe how a derivative function $f'(x)$ relates to the function $f(x)$. (There is no specific formula for $f(x)$ here. What characteristics of the function $f(x)$ are described by the derivative function $f'(x)$?)
4.) (15 pts.) Given the graph of \( f(x) \) below, graph an antiderivative \( F(x) \) on the bottom left set of axes, and graph \( f'(x) \) on the bottom right set of axes.
5.) (15 pts.)

a.) (5 pts.) Given a function \( f(x) \), write the definition of \( f'(x) \). (This is a formula.)

b.) (10 pts.) For \( f(x) = 2x^2 - 3 \), use your definition in part (a) to compute \( f'(x) \). (Note: applying the power law or similar rules does not earn any credit for this problem. You need to use the definition, along with algebra.)
6.) (15 pts.) Let \( g(t) = \frac{3}{t} - \sqrt{t - 5}. \)

a.) (5 pts.) Create and use a table of values to approximate \( g'(10) \) numerically.

b.) (5 pts.) Compute \( g'(t) \) using any method you prefer.

c.) (5 pts.) Use your formula from part (b) to evaluate \( g'(10) \) exactly. How close was your approximation from part (a)?
7.) (15 pts.)

a.) (5 pts.) Find the general solution to the differential equation \( f'(x) = 3x \).

b.) (3 pts.) How many solutions are there to the differential equation \( f'(x) = 3x \)?

c.) (4 pts.) Find the unique solution to the initial value problem \( f'(x) = 3x, \, f(2) = 10 \).

d.) (3 pts.) How many solutions are there to the initial value problem \( f'(x) = 3x, \, f(2) = 10 \)?

BONUS: (5 pts.) Write an original poem about a calculus concept. You may submit your poem on a separate sheet of paper, or write it here or on the back of this page.