NAME:

Show ALL your work CAREFULLY.

(a) Use the technique of integration by parts to find the following indefinite integral

\[ \int x e^{-2x} \, dx. \]

Let \( u = x \) and \( dv = e^{-2x} \, dx \). It follows that \( du = dx \) and \( v = -\frac{e^{-2x}}{2} \). Thus,

\[ \int x e^{-2x} \, dx = x \left( -\frac{e^{-2x}}{2} \right) - \int \left( -\frac{e^{-2x}}{2} \right) \, dx \]

\[ = -\frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4} + C \]

\[ = -\frac{e^{-2x}(2x + 1)}{4} + C. \]

(b) Use the technique of partial fractions to find the following indefinite integral.

\[ \int \frac{dx}{x^2 - x - 2}. \]

First, we wish to write

\[ \frac{1}{x^2 - x - 2} = \frac{1}{(x - 2)(x + 1)} = \frac{A}{x - 2} + \frac{B}{x + 1} \]

for some constants \( A \) and \( B \).

By equating the numerators of both sides, we must have

(1) \( 1 \equiv A(x + 1) + B(x - 2). \)

At \( x = -1 \), (1) becomes \( 1 = 0 + B((-1) - 2). \) It follows that \( B = \frac{-1}{3} \). At \( x = 2 \), (1) becomes \( 1 = A((2) + 1) + 0. \) It follows that \( A = \frac{1}{3} \). Now,

\[ \int \frac{dx}{x^2 - x - 2} = \int \frac{(\frac{1}{3})}{x - 2} + \frac{(-\frac{1}{3})}{x + 1} \, dx \]

\[ = \frac{1}{3} \int \frac{dx}{x - 2} - \frac{1}{3} \int \frac{dx}{x + 1} \]

\[ = \frac{1}{3} (\ln |x - 2| - \ln |x + 1|) + C. \]