Math 205 Quiz 4
Name:

1. Let $A$ be an $m \times n$ matrix, $B$ be an $n \times p$ matrix, and $C$ be an $n \times m$ matrix where $n \neq m \neq p$.

   (a) Circle the matrix multiplications that can be computed.

   | $AB$ | $BC$ | $CB$ | $CA$ | $AC$ | $A^T B$ | $A^T C$ | $B^T C$ |

2. Consider the matrix $A = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 4 & 0 \\ 0 & -4 & 1 \end{pmatrix}$.

   (a) Find the inverse of $A$.

   \[
   \begin{pmatrix} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 1 & 0 \\ 0 & -4 & 1 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_3=R_2+R_3} \begin{pmatrix} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{1/4*R_2} \begin{pmatrix} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/4 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}
   
   \]

   \[
   \begin{pmatrix} 1 & 0 & 0 & 1 & 1/2 & 0 \\ 0 & 1 & 0 & 0 & 1/4 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{R_1=2*R_2+R_1} \text{then } A^{-1} = \begin{pmatrix} 1 & 1/2 & 0 \\ 0 & 1/4 & 0 \\ 0 & 1 & 1 \end{pmatrix}
   
   (b) Row reducing $A$ to the Identity can be done in three steps. Therefore there are three elementary matrices, $E_1$, $E_2$, and $E_3$ such that $E_3E_2E_1A = I$. Determine these matrices. Order matters. $E_1$ is the first row operation to be performed, etc.

   \[
   E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad E_3 = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
   
   \]

   These are not unique. If you did a different order or different operations, then you have different $E_1, E_2, E_3$.

3. Consider the matrix $A = \begin{pmatrix} 4 & h \\ h & 1 \end{pmatrix}$.

   (a) For what value(s) of $h$ will $A^{-1}$ fail to exist?

   \[
   \det(A) = 4 - h^2 \text{ and the determinant cannot be zero. So, } h \neq \pm 2.
   
   (b) Determine the inverse of the matrix $A$ when it exists.

   \[
   \frac{1}{4-h^2} \begin{pmatrix} 1 & -h \\ -h & 4 \end{pmatrix}
   
   1