Math 105: Review for Exam I

1. Let \( f(x) = 3 + \sqrt{x} + 5. \)
   (a) What is the natural domain of \( f \)?
   (b) What is the range of \( f \)?

2. For the graph of \( f \) shown, answer the following.
   (a) Evaluate the following.
      i. \( f'(-2) \)
      ii. \( f(3) \)
      iii. \( \lim_{x \to 3^-} f(x) \)
      iv. \( \lim_{x \to 3^+} f(x) \)
      v. \( \lim_{x \to 3} f(x) \)
      vi. \( \lim_{x \to 2} f(x) \)
   (b) Where is \( f \) discontinuous?
   (c) Where does \( f' \) fail to exist?

3. Let \( f(x) = 3x^2 - 2x. \)
   (a) Compute the average rate of change of \( f \) on the interval \([2, 2.1]\).
   (b) Using the limit definition of the derivative, find \( f'(x) \).
   (c) Find the equation of the tangent line to \( f \) at \( x = 2 \).
   (d) How would the derivative of \( g(x) = f(x) + 5 \) compare to \( f'(x) \)?
   (e) How would the derivative of \( h(x) = 5f(x) \) compare to \( f'(x) \)?
4. Fill in the table showing the graphical relationships between \( f \), \( f' \), and \( f'' \).

<table>
<thead>
<tr>
<th></th>
<th>positive</th>
<th>negative</th>
<th>increasing</th>
<th>decreasing</th>
<th>concave up</th>
<th>concave down</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
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<td></td>
<td></td>
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<tr>
<td>( f' )</td>
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<td></td>
<td></td>
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<tr>
<td>( f'' )</td>
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</tbody>
</table>

5. Given the graph of \( f \), sketch a graph of \( f' \) and a graph of \( F \), an antiderivative of \( f \) such that \( F(0) = -2 \).

![Graph of f, f', and F](image)

6. Shown below is a graph of \( f' \) on its entire domain. The graph is NOT \( f \).

At which \( x \)-value(s)
- (a) does \( f \) have a stationary point?
- (b) \( f \) decreasing?
- (c) \( f' \) increasing?
- (d) \( f' \) decreasing?
- (e) \( f \) concave up?
- (f) \( f \) concave down?
- (g) is \( f \) greatest?
- (h) is \( f \) least?
- (i) is \( f' \) greatest?
- (j) is \( f' \) least?
- (k) is \( f'' \) greatest?
- (l) is \( f'' \) least?

On what interval(s) is
- (a) \( f \) increasing?
7. Suppose that $T(t)$ gives the temperature in Lewiston as a function of time. In each of the following situations, determine if the signs of $T$, $T'$, and $T''$ are positive, negative, zero, or unknown.

(a) The temperature is 60 degrees and falling steadily.

(b) The temperature is rising more and more slowly.

(c) The temperature is $-5$ degrees and rising.

8. The table below gives some values for a function $f(x)$ whose derivative exists at all $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
<th>1.1</th>
<th>1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>5.0</td>
<td>6.2</td>
<td>7.3</td>
<td>8.2</td>
<td>9.0</td>
</tr>
</tbody>
</table>

(a) Estimate $f'(1.05)$.

(b) Based on the data, is $f''(1.0)$ positive or negative?

9. Find the derivatives of the following.

(a) $y = 2 + 3x + x^4 + 5x^6$

(b) $y = \sqrt[3]{x} + \frac{1}{x^6} + \frac{x}{6} + \frac{\pi}{6} + 6^{1/2}$
10. Find antiderivatives of the following.
   (a) \( y = \pi + 3x^2 \)
   
   (b) \( y = 4x^5 - \frac{1}{x^6} \)

11. Is \( y = 5x^3 \) a solution to the differential equation \( xy' - 3y = 0 \)? [Students in the 8:00 and 9:30 sections may omit this problem.]

12. Solve the IVP (initial value problem) \( 1 = x^3 - y'(x) \) if \( y(2) = 13 \). [Students in the 8:00 and 9:30 sections may omit this problem.]

See old exams and quizzes at http://abacus.bates.edu/~etowne/mathresources.html