There are 6 problems in this exam. On each problem, you must show all your work, or otherwise thoroughly explain your conclusions. **There is always at least one step preceding a final answer.** Units may be requested for your final answer; a point deduction will apply if they are omitted.

On the portion of the exam marked **No Calculator**, you will be allowed 20 minutes during which your calculator must be closed and put away. If you finish this section early, you may hand in your work early. However, **only after you hand in the "no calculators" section will you be permitted to use a calculator.**

You will have 55 minutes to complete this exam.

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Problem 1-NC. (20 points) Consider the function $h$ defined by the formula

$$h(x) = \frac{x^4 + \sqrt{x}}{x}.$$ 

(a) (10 points) What is the natural domain of $h$? Why?

Any $x$ will work provided:

- $x$ is not negative (since then $\sqrt{x}$ will be nonreal)
- $x$ is not zero (since then division by $x$ is undefined.)

Thus $(0,\infty)$ or $\{ x \in \mathbb{R} \mid x > 0 \}$ is the domain of $h$.

(b) (10 points) Find an antiderivative $H$ which satisfies $H(1) = \frac{1}{4}$.

Hint: You might want to perform some algebra before doing any calculus.

$$h(x) = \frac{x^4 + \sqrt{x}}{x} = \frac{x^4}{x} + \frac{\sqrt{x}}{x} = x^3 + x^{-\frac{1}{2}}$$

$$H(x) = \frac{x^4}{4} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$H(x) = \frac{x^4}{4} + 2\sqrt{x} + C.$$ 

$$\frac{1}{4} = \frac{1}{4} + 2\sqrt{1} + C$$

$$\frac{1}{4} = \frac{1}{4} + 2 + C$$

$$-2 = C.$$ 

Thus $H(x) = \frac{x^4}{4} + 2\sqrt{x} - 2.$
Problem 2-NC. (30 points) In this problem, you will ultimately sketch a graph of the function

\[ y(t) = 2t^3 - \frac{9}{2}t^2 - \frac{1}{4}t^4. \]

To help with your analysis, you are told that all of the important changes in the graph of \( y \) occur at one of the points \( t = 0, t = 1, t = 2, \) or \( t = 3. \)

(a) (10 points) Which of these points is/are stationary points of \( y \)? Determine whether each is a local maximum, local minimum, or neither.

\[ y'(t) = 6t^2 - 9t - t^3 \]

\[ y'(0) = 0 \text{ so } t=0 \text{ is stationary.} \]

\[ y'(1) = -4 \quad y'(2) = -2 \]

\[ y'(3) = 0 \text{ so } t=3 \text{ is stationary} \]

Check \( y'' \) at each.

\[ y''(t) = 12t - 9 - 3t^2 \]

\[ y''(0) = -9 \text{ so } t=0 \text{ is local max.} \]

\[ y''(3) = 0 \text{ so } t=3 \text{ is neither.} \]

(b) (10 points) Which of these points is/are inflection points of \( y \)?

Check \( y''(t) = 0. \)

\[ y''(0) = -9 \]

\[ y''(1) = 0 \text{ so } t=1 \text{ is inflection} \]

\[ y''(2) = 3 \]

\[ y''(3) = 0 \text{ so } t=3 \text{ is inflection} \]

(c) (10 points) Sketch a graph of \( y(t) \) on the axes provided. Worry only about the shape of the graph — direction and concavity — and not the vertical scale. Note, however, that \( y(0) = 0. \)
Problem 1. (20 points) An electrocardiogram (EKG) is a record of the amplitude $A$ of electrical activity in the heart as a function of time $t$. Shown below is an EKG of a (mostly) healthy human heart.

(a) (5 points) Which of the following words describes the function $A$? Circle all that apply.

- periodic
- even
- odd
- continuous
- polynomial
- exponential
- logarithmic
- constant

(b) (5 points) Compare and contrast the slope of $A(t)$ between points $Q$ and $R$ and the slope between points $R$ and $S$.

- Constant on both $QR$ and $RS$
- Positive on $QR$ but negative on $RS$
- Steeper (larger in absolute value) on $RS$ than $QR$

(c) (5 points) Circle one:

- $A'(P)$ is... negative zero positive
- $A''(P)$ is... negative zero positive

What do your answers imply about the function $A$ at the point $P$?

$A$ has a local maximum at $P$.

(d) (5 points) Explain what $A'(R)$ is, using the word “limit” in your answer.

It appears as though $A'(R)$ does not exist, since its limit (the slope) on the left is positive and on the right is negative.

These two do not agree, so $A'(R)$ does not exist.
Problem 2. (25 points) Inside a mystery box is a differentiable function $f$ whose derivative $f'$ is graphed below.

(a) (6 points) On what interval(s) is $f$ an increasing function?

$(-3,-2) \cup (-2,3) \cup (6,7)$

($f'$ positive)

(b) (6 points) On what interval(s) is $f$ concave up?

$(-2,1) \cup (5,7)$

($f'$ increasing)

(c) (6 points) What kind of stationary point does $f$ have at $x = -2$? Why?

Neither max nor min — $f'$ remains positive on both sides, so $f$ remains increasing on both sides.

(d) (7 points) Using the axes provided, sketch a graph of the function $f$. 

[Graph of $f$ provided]
Problem 3. (25 points) You’ve just brought two cats home from an animal shelter, and quickly realize they hate each other. Their happiness $h$ is a function of how close they are to one another $r$, measured in feet, according to the function

$$h(r) = \frac{1}{r^8} - \frac{80}{r^{10}} = r^{-8} - 80r^{-10}.$$ 

(a) (10 points) Verify that $h(r)$ satisfies the differential equation

$$h' = -\frac{8h}{r} + \frac{160}{r^{11}}.$$

$$h'(r) = -8r^{-9} - 80(-10)r^{-11}$$

$$= -8r^{-9} + 800r^{-11}.$$ 

$$\frac{-8h}{r} = -\frac{3r^{-8} + 640r^{-10}}{r}$$

$$= -8r^{-9} + 640r^{-11}$$

$$+ \frac{160}{r^{11}} = +160r^{-11}$$

$$= -8r^{-9} + 800r^{-11}.$$ 

Thus $h'(r) = \frac{-8h}{r} + \frac{160}{r^{11}}$.

(b) (15 points) What distance apart makes the cats happiest? (Use the derivative you computed in (a).)

Find a local max — or at least a stationary pt. Where is $h'=0$?

$$h'(r) = -8r^{-9} + 800r^{-11} = 0$$

Multiply by $-\frac{1}{8}r^{11}$:

$$r^2 + 100 = 0$$

$$r^2 = 100$$

$$r = \pm 10$$

The cats are happiest 10 feet apart.
Problem 4. (30 points) Because free fall just isn’t enough for some people, and terminal velocity is “so last century,” the newest craze in extreme sports is jetpack skydiving, where a jetpack is used to accelerate the skydiver toward the ground.

On her most recent jetpack skydive, Eve recorded her velocity during the dive to impress her friends. Upon arriving home, she fit the polynomial function

\[ v(t) = 66t - 11t^2 \]

to her velocity, where \( v \) is measured in meters/second and \( t \) is in minutes.

(a) (15 points) Use a difference quotient with \( h = 0.01 \) to estimate \( v'(1) \), her acceleration after 1 minute. Include units in your answer.

\[
\begin{align*}
v'(1) & = \lim_{h \to 0} \frac{v(1+h) - v(1)}{h} \\
& \approx \frac{v(1.01) - v(1)}{0.01} \\
& = \frac{0.4389 \text{ m/s}}{0.01 \text{ min}} \\
& = 43.89 \text{ m/s/min} \text{ or } \frac{43.89}{60} \approx 0.7315 \text{ m/s}^2.
\end{align*}
\]

(b) (15 points) By finding her distance function \( d(t) \), determine how many meters Eve travelled during the first 3 minutes of her jetpack skydive.

Hint: You may assume \( d(0) = 0 \).

The antiderivative of \( v(t) \)

\[
d(t) = 66 \frac{t^2}{2} - 11 \frac{t^3}{3} = 33t^2 - \frac{11}{3}t^3 + C
\]

\[
d(3) = 33(3)^2 - \frac{11}{3}(3)^3
\]

\[
= 198. \quad \text{Really this ought to be } 198 \times 60 = 11880 \text{ m, but this answer is acceptable.}
\]