Read directions carefully and show all your work. Partial credit will be assigned based upon the correctness, completeness, and clarity of your answers. In other words, correct answers accompanied by incorrect or incomplete work will not receive full credit.

1. (30 pts) Evaluate the following integrals.

   (a) \( \int \sin x \cos x \, dx \)

   (b) \( \int x \sqrt{10 - x} \, dx \)

   (c) Suppose \( \int_1^2 f(u) \, du = 5 \), evaluate \( \int_0^{1/2} f(1 + 2t) \, dt \).
2. (17 pts) Consider a truncated conical container full of strawberry milkshake. The cup is 7 inches tall, 2.5 inches across the base, 3.5 inches across the top, and with a straw sticking up 1 inch above the top of the cup. This kind of strawberry milkshake weighs $\frac{4}{9}$ oz./in$^3$. **Set up, but do not evaluate** an integral representing the work done in sucking all of the milkshake out of the straw.
3. (12 pts) Suppose the graph of a function $f$ is increasing and concave down on the interval $[a, b]$. List the following quantities in increasing order: $L_{50}$, $R_{50}$, $M_{50}$, $T_{50}$, $\int_a^b f(x) \, dx$. Be sure to provide a sentence or two explaining your reasons for the ordering.

4. (17 pts) Use separation of variables to solve the following initial value problem:

$$\frac{dy}{dt} = \frac{e^t}{y} \quad \text{with} \quad y(1) = 2\sqrt{e}. $$
5. Consider the region bounded by \( y = \sqrt{x^3}, \ y = 1, \) and the \( y\)-axis.

(a) (12 pts) Which of the following represents an integral expression for the region described above? SELECT ALL THAT APPLY.

i. \( \int_0^1 \sqrt{x^3} \, dx \)

ii. \( \int_0^1 \left( 1 - \sqrt{x^3} \right) \, dx \)

iii. \( \int_0^1 \sqrt[3]{y^2} \, dy \)

iv. \( \int_0^1 \pi \left( \sqrt{x^3} \right)^2 \, dx \)

(b) (6 pts) Which of the following represents an integral expression for the volume of the solid created when region described above is rotated around the line \( y = 1 \)?

i. \( \pi \int_0^1 \left( \sqrt[3]{y^2} \right)^2 \, dy \)

ii. \( \pi \int_0^1 \left( 1 - \left( \sqrt[3]{y^2} \right)^2 \right) \, dy \)

iii. \( \pi \int_0^1 \left( 1 - \sqrt{x^3} \right)^2 \, dx \)

iv. \( \pi \int_0^1 \left( \sqrt{x^3} - 1 \right)^2 \, dx \)

v. \( \pi \int_0^1 \left( \sqrt{x^3} \right)^2 \, dx \)

(c) (6 pts) Which of the following represents an integral expression for the volume of the solid created when region described above is rotated around the line \( x = -2 \)?

i. \( \pi \int_0^1 \left[ \left( 2 + \sqrt[3]{y^2} \right)^2 - 4 \right] \, dy \)

ii. \( \pi \int_0^1 \left[ \left( 2 + \sqrt[3]{y^2} \right) - 2 \right]^2 \, dy \)

iii. \( \pi \int_0^1 \left[ \left( \sqrt{x^3} + 2 \right)^2 - 4 \right] \, dx \)

iv. \( \pi \int_0^1 \left[ 4 - \left( \sqrt{x^3} + 2 \right)^2 \right] \, dx \)