Please show your work.

1. The graph of a function, f(x), is increasing and concave down on the interval [a, b]. Put the following quantities in increasing order:

\[ L_{100}, R_{100}, T_{100}, M_{100}, \int_a^b f(x) \, dx. \]

2. Evaluate by finding an antiderivative. [Your final answer should not contain an integral.]

a) \[ \int (x^2 - 9)^3 \, x \, dx \]

b) \[ \int \frac{\cos(x)}{1 - \sin(x)} \, dx \]

3. Evaluate: \[ \int_0^1 \frac{1}{x^2 - 4x + 4} \, dx \]
4. A cylindrical tank of radius 4 ft. and height 10 ft is filled halfway with water. Find the work required to pump all the water over the upper rim.

5. Find the arclength of a function on the interval [2,3] where the derivative of the function is given by

\[ f'(x) = \sqrt{x^2 - 1}. \]
6. Consider the functions \( f(x) = \frac{x^2}{4} \) and \( g(x) = x \).

a) Draw the graphs of \( g(x) \) and \( f(x) \).

b) Using integration, find the area between the two functions.

c) Find the volume of the solid formed by revolving the area between the functions around the x-axis.
7. Find the solution of the initial value problem:

\[
\frac{dy}{dx} = x(1 + y^2) \quad \text{with} \quad y(0) = 1.
\]

8. Let \( I = \int_{0}^{1} \frac{4}{1 + x^2} \, dx \). If we were to calculate the integral exactly, we would find that \( I = \pi \). Therefore, we can use this result and numerical integration techniques to approximate the value of \( \pi \).

a) Compute \( L_4 \), the left sum approximation with 4 subdivisions.

b) How many subdivisions are required to obtain a left sum approximation with error of at most 1/10,000? Recall that the error bound estimate for left sums may be determined using

\[
|I - L_n| \leq \frac{K(b - a)^2}{2n}.
\]