Math 105: Review for Exam I

1. Let \( f(x) = 3 + \sqrt{x + 5} \).
   (a) What is the natural domain of \( f \)?
   (b) What is the range of \( f \)?

2. For the graph of \( f \) shown, answer the following.
   (a) Evaluate the following.
      i. \( f'(-2) \)
      ii. \( f(3) \)
      iii. \( \lim_{x \to 3^-} f(x) \)
      iv. \( \lim_{x \to 3^+} f(x) \)
      v. \( \lim_{x \to 3} f(x) \)
      vi. \( \lim_{x \to 2} f(x) \)
   (b) Where is \( f \) discontinuous?
   (c) Where does \( f' \) fail to exist?

3. Let \( f(x) = 3x^2 - 2x \).
   (a) Compute the average rate of change of \( f \) on the interval \([2, 2.1]\).
   (b) Using the limit definition of the derivative, find \( f'(x) \).
   (c) Find the equation of the tangent line to \( f \) at \( x = 2 \).
   (d) How would the derivative of \( g(x) = f(x) + 5 \) compare to \( f'(x) \)?
   (e) How would the derivative of \( h(x) = 5f(x) \) compare to \( f'(x) \)?
4. Given the graph of $f$, sketch a graph of $f'$ and a graph of $F$, an antiderivative of $f$ such that $F(0) = -2$.

5. Shown below is a graph of $f'$ on its entire domain. The graph is NOT $f$.

At which $x$-value(s) does $f$ have:
(a) a stationary point?
(b) a local max?
(c) a local min?
(d) a stationary point?
(e) a local max?
(f) a local min?

On what interval(s) is:
(a) $f$ increasing?
(b) $f$ decreasing?
(c) $f'$ increasing?
(d) $f'$ decreasing?
(e) $f$ concave up?
(f) $f$ concave down?
6. Suppose that $T(t)$ gives the temperature in Lewiston as a function of time. In each of the following situations, determine if the signs of $T$, $T'$, and $T''$ are positive, negative, zero, or unknown.

(a) The temperature is 60 degrees and falling steadily.

(b) The temperature is rising more and more slowly.

(c) The temperature is $-5$ degrees and rising.

7. The table below gives some values for a function $f(x)$ whose derivative exists at all $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
<th>1.1</th>
<th>1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>5.0</td>
<td>6.2</td>
<td>7.3</td>
<td>8.2</td>
<td>9.0</td>
</tr>
</tbody>
</table>

(a) Estimate $f'(1.05)$.

(b) Based on the data, is $f''(1.0)$ positive or negative?

8. Find the derivatives of the following.

(a) $y = 2 + 3x + x^4 + 5x^6$

(b) $y = \sqrt{x} + \frac{1}{x^6} + \frac{x}{6} + \frac{\pi}{x} + 6^{1/2}$

9. Find antiderivatives of the following.

(a) $y = \pi + 3x^2$

(b) $y = 4x^5 - \frac{1}{x^6}$