1. Let \( g(w) = w^3 + w \). Evaluate \( \lim_{w \to 1} \frac{g(w) - 2}{w - 1} \).

Note that the limit we want to evaluate resembles the limit of a difference quotient. Indeed, \( g(1) = 2 \) so that
\[
\lim_{w \to 1} \frac{g(w) - 2}{w - 1} = \lim_{w \to 1} \frac{g(w) - g(1)}{w - 1} = g'(1).
\]

Now, \( g'(w) = 3w^2 + 1 \) and thus \( g'(1) = 4 \). Hence, \( \lim_{w \to 1} \frac{g(w) - 2}{w - 1} = 4 \).

2. The graph of a function \( F \) is shown below. (a) Find, if it exists, each of the following limits:

(i) \( \lim_{x \to 1^+} F(x) \). This limit is 0.

(ii) \( \lim_{x \to 1} F(x) \). This limit exists and is equal to 2.

(b) Is \( F \) continuous at \( x = -1 \)? Explain. No, since \( F(-1) = 1 \neq 2 = \lim_{x \to -1} F(x) \).

(c) Suppose there is a function \( f \) such that \( F(x) = \frac{f(x) - f(3)}{x - 3} \). Find \( f'(3) \), if it exists.

Note that \( f'(3) \) is the limit of the difference quotient \( \frac{f(x) - f(3)}{x - 3} \). In other words, \( f'(3) = \lim_{x \to 3} F(x) \). From the given graph of \( F \), we see that this limit exists and is equal to 2. Thus, we have \( f'(3) = 2 \).

Date: October 1, 2010.