1. Consider the function $f$ shown above. Next to it, sketch the graph of $A_f(x) = \int_2^x f(t) \, dt$. Make sure the slopes on your graph are correct. (The curved portion of $f$ is one-quarter of a circle of radius 1 centered at (5,0)).

2A. Use the method of substitution to find the following: $\int_0^1 \frac{e^{-2x}}{\sqrt{1 + e^{-2x}}} \, dx$. Show all your steps. Express the answer as a decimal number to four places after the decimal.

2B. Just to be clear: What are the new limits (written to four decimal places) on the integral after the substitution is made?
3. The region $S$ below is bounded by the graphs of $y = 1 + \sqrt{x}$, $y = \frac{4}{5}(x - 4)$ and $x = 4$.

*Hint:* One of the following problems may require *two* separate integrals!

3A. Suppose the region $S$ is rotated around the line $y = 10$. Set up the integral (or integrals) giving the exact volume of the resulting solid of revolution.

3B. Suppose the region $S$ is rotated around the $y$ axis. Set up the integral (or integrals) giving the exact volume of the resulting solid of revolution.
4. Suppose a trough shaped like the one to the right is filled with water to the one-foot mark. The trough measures 8 feet long, and cross sections are bounded by the graphs of $y = x^2$ and $y = 4$. Water weighs 62.4 pounds per cubic foot.

4A. In terms of $y$, what is the (approximate) volume of a thin sheet of water at a height of $y_i$ feet up from the bottom of the tank and “thickness” $\triangle y$?

4B. In terms of $y$, what is the distance that sheet of water has to travel if the water is to be pumped to a point 15 feet above the top of the trough?

4C. What integral represents how much work is done against gravity in pumping out all the water to a point 15 feet above the top of the trough? (Just set it up; no need to evaluate it).
5A. Let $I$ be the exact value of $\int_{a}^{b} f(x) \, dx$ where $f$ is some continuous function on $[a, b]$. "Theorem 3" (from section 6.2) says that the error committed by MID($n$) in approximating $I$, that is, $|I - \text{MID}(n)|$, is smaller than what expression? (Be sure to say what $K_2$ means in your answer).

5B. Now let $f(x) = \frac{1}{12}x^4 - \frac{2}{3}x^3 - 2x^2$; then $f'(x) = \frac{1}{3}x^3 - 2x^2 - 4x$ and $f''(x) = x^2 - 4x - 4$.

The exact value of $\int_{1}^{4} f(x) \, dx$ is $-1349/20 = -67.45$; you do not have to check this.

What is the smallest value of $n$ for which theorem 3 guarantees $|I - \text{MID}(n)| < 0.0005$? Show your calculations. (Use the best possible $K_2$).

5C. For your value of $n$ in the previous problem, what is MID($n$) and how far is it from the exact value? Write both answers to six places after the decimal point.

MID($n$) equals? the error is?

5D. What is TRAP(30) for this integral? Show any intermediate values needed to find TRAP(30).