Read directions carefully and show all your work. Partial credit will be assigned based upon the correctness, completeness, and clarity of your answers.

1. Evaluate the following integrals.

   (a) \[ \int_{0}^{\sqrt{\pi}} x^2 \cos(x^3) \, dx \]

   (b) \[ \int_{0}^{1/2} \frac{\arcsin x}{\sqrt{1-x^2}} \, dx \]
2. Suppose 10 foot long fuel tank on a truck has trapezoidal cross sections where the base of the trapezoid is 2 feet wide, the top is 4 feet wide, and the height is 8 feet, as shown in the figure. Assume that an engine is approximately 3 feet above the top of the fuel tank and that diesel fuel weighs approximately 55.6 pounds per cubic foot. Set up, but do not evaluate, the integral that finds the amount of work done by the fuel pump in raising fuel to the level of the engine when the tank is half full.
3. Consider the initial value problem \( \frac{dy}{dx} = y \ln y \) with \( y(1) = e \). Use separation of variables to find the solution to the IVP.

4. Let \( I = \int_0^1 2x^3 \, dx \). How many subdivisions are required to obtain a trapezoidal sum approximation with error of at most \( 1/10,000 \)? Recall that the error bound estimates for trapezoidal sums may be determined using:

\[
|I - T_n| \leq \frac{K_2(b-a)^3}{12n^2}.
\]
5. Consider the region bounded by the curves \( x = 3y^2 - 2 \) and \( x = y^2 \) and the line \( y = 0 \).

(a) Set up, but do not evaluate, the integral which represents the volume of the solid formed by revolving the region around the \( x \)-axis.

(b) Set up, but do not evaluate, the integral which represents the volume of the solid formed by revolving the region around the line \( x = 1 \).
6. Consider a function \( f \) where the graph of its derivative \( f' \) and tables of values is shown below.

\begin{center}
\begin{tabular}{c|c|c|c|c|c}
\hline
\( x \) & 0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0 \\
\hline
\( f'(x) \) & 0.5 & 0.54 & 0.659 & 0.852 & 1.097 & 1.342 \\
\hline
\end{tabular}
\end{center}

(a) Write an integral that represents the arc length of \( f \) on the interval \([0, 1]\).

(b) Use the table of values above to approximate your integral from (a) using \( R_5 \). Round to 3 digits after the decimal. Note: you might want to expand the table above by adding a row for values of the integrand.

(c) Determine if \( R_5 \) is an overestimate or underestimate for the exact value of the integral in (a). Justify your answer. (Hint: carefully consider the integrand in part (a) as well as characteristics about \( f' \).)