Math 106: Review for Exam I

1. Find the following. [Substitution tip: usually let $u$ = a function that’s “inside” another function, especially if $du$ (possibly off by a multiplying constant) is also present in the integrand.]

(a) \[ \int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx \]

(b) \[ \int_{\pi}^{2\pi} \cos^7(5x) \sin(5x) \, dx \]

(c) \[ \int \frac{7x^2}{1 + x^6} \, dx \]

(d) \[ \int_6^{10} x \sqrt{10 - x} \, dx \]
2. Suppose \( f(x) \) is decreasing and concave up.
   (a) Put the following quantities in ascending order.
   \[ L_{100}, R_{100}, T_{100}, M_{100}, \int_a^b f(x) \, dx \]
   (b) What can you say with certainty about where \( S_{200} \) would fit into your list above?

3. Suppose \( f(t) \) is the rate of change (in animals per month) of a population \( P(t) \).
   (a) What does \( \int_4^{12} f(t) \, dt \) represent in this problem?
   (b) Find the best possible left, right, midpoint, trapezoidal, and Simpson’s approximations to \( \int_4^{12} f(t) \, dt \)
       given the data in the table below.

<table>
<thead>
<tr>
<th></th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>15</td>
<td>11</td>
<td>8</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

4. Find bounds for each of the following errors if \( I = \int_2^7 \ln x \, dx \).
   (a) \( |I - L_{100}| \)
   (b) \( |I - T_{100}| \)
   (c) \( |I - M_{100}| \)

5. If \( I = \int_2^7 \ln x \, dx \), how many subdivisions are required to obtain a trapezoidal sum approximation with
   error of at most \( 1/1,000,000 \)?
6. Use Euler’s method with three steps on the differential equation \( \frac{dy}{dt} = y - t \) to estimate \( y(2.5) \) if \( y(1) = 0 \).

7. Solve the differential equation \( \frac{dy}{dx} = 2xy + 6x \) if the solution passes through \( (0, 5) \).

8. Write integrals equal to

   (a) the arc length of \( y = x^2 \) on the interval \([1, 5]\)

   (b) the area bounded by \( y = x^2 - 8x + 24 \) and \( y = 3x \)
9. Consider the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 9$. Write an integral equal to the volume generated if this region is rotated about

(a) the $x$-axis

(b) the line $x = -1$

10. A pyramid has a square base 30 feet to a side and a height of 10 feet. Write integrals equal to

(a) the volume of the pyramid

(b) the work done in pumping all the fluid to a point 5 feet above the pyramid if the pyramid is filled to a height of 8 feet with water (62.4 pounds per cubic foot)