Math 206 Test One

Name: ______________________

1. (5 marks, level of difficulty 5) Answer one of a. or b.

a. Prove that if \( f : U \subset \mathbb{R}^n \rightarrow \mathbb{R} \) is continuous at \( \vec{a} \) and \( f(\vec{a}) \neq b \) for some number \( b \), then there exists \( r > 0 \) such that \( f(\vec{x}) \neq b \) for all \( \vec{x} \in B_r(\vec{a}) \).

b. Use analysis to prove that \( \lim_{(x,y) \to (0,0)} \frac{y^2}{\sqrt{x^2+y^2}} = 0 \)
2. (5 marks, level of difficulty 1) Answer one of a. or b.

a. Identify $C = \{(x, y) : -5 < x < 5\} \subset \mathbb{R}^2$ as open, closed, or neither and find the boundary and complement.

b. Find (approximately) the maximum and minimum values of the function $h(x, y) = \sqrt{x^2 + y^2}$ on the set $S = \{(x, y) : x^2 + y^2 \leq 4\}$. 
3. (5 marks, level of difficulty 2) Answer one of a. or b.

a. Show that \( f(x, y) = e^{x-y} \) is a solution to the partial differential equation \( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \).

b. Compute \( \frac{\partial}{\partial x} (xe^y) \) using the Definition of the partial derivative.
4. (5 marks, level of difficulty 3) Answer one of a. or b.

a. Calculate the Jacobian matrix for the function \( f(x, y) = 8x - 7y + 2 \) at the point \( \vec{a} = (-4, -5) \). Then write a formula for the total derivative.

b. Show that if \( f : U \subset \mathbb{R}^n \to \mathbb{R}^n \) is a linear transformation, then \( (Df(\vec{a}))(\vec{x}) = f(\vec{x}) \).
5. (5 marks, level of difficulty 4) Answer one of a. or b.

a. Use the chain rule to find the derivative of \( \vec{g} \circ \vec{f} \) at the point \( \vec{a} = (3, 2) \) where \( \vec{g}(x, y) = (x^2y^3, 3x - y^2) \) and \( \vec{f}(x, y) = (-y, x) \).

b. Find \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \) assuming that the equation \( x^3y^2z + xy - z^3 = 0 \) implicitly defines \( z \) as a function of \( x \) and \( y \).