Math 105 Section C
Quiz 2 (10 points)

Name: Solutions

- Show all your work to receive full credit for a problem.

1. Find the derivatives of the following functions. You may use only the power rule, the sum rule and the constant multiple rule for derivatives.

   (a) \( g(t) = \frac{3t^7}{4} - \frac{11}{5t^2} + 10 \)

   \[ g(t) = \frac{3}{4} t^7 - \frac{11}{5} t^{-2} + 10 \]

   \[ g'(t) = \frac{3}{4} \cdot 7t^6 - \frac{11}{5} \cdot (-2) t^{-3} + 0 \]

   So \( g'(t) = \frac{21}{4} t^6 + \frac{22}{5} t^{-3} \)

   (b) \( f(x) = \sqrt{\frac{2}{x^3}} + 100x \)

   \[ f(x) = \sqrt{2} \cdot x^{-3/2} + 100 x = \sqrt{2} \cdot x^{3/2} + 100 x \]

   \[ f'(x) = \sqrt{2} \cdot \frac{-3}{2} x^{-5/2} + 100 \]

   i.e. \( f'(x) = \frac{-3 \sqrt{2}}{2x^{5/2}} + 100 \)
2. Let \( f(x) = \frac{3}{x} \). Use this function to answer the questions that follow.

(a) Use the limit definition of the derivative to find \( f'(2) \).

\[
\begin{align*}
f'(2) &= \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} \\
&= \lim_{h \to 0} \frac{\frac{3}{2+h} - \frac{3}{2}}{h} \\
&= \lim_{h \to 0} \frac{6-6-3h}{4+2h} \\
&= \lim_{h \to 0} \frac{-3h}{4+2h} \\
&= \lim_{h \to 0} \frac{-3}{4+2h} = -\frac{3}{4}
\end{align*}
\]

(b) Find an equation of the line tangent to the graph of \( f \) at \( x = 2 \).

\[
f(2) = \frac{3}{2}, \quad f'(2) = -\frac{3}{4} = \text{slope of the tangent line}
\]

So equation of the tangent line is

\[
y - \frac{3}{2} = -\frac{3}{4} (x-2)
\]

\[
y - \frac{3}{2} = -\frac{3}{4} x + \frac{3}{2}
\]

\[
\therefore \quad y = -\frac{3}{4} x + 3
\]