1. Consider the linear transformation $T(x_1, x_2, x_3) = (x_1 - 2x_2, x_1 + x_2 + x_3)$.

   (a) What is the domain of $T$?

   (b) What is the codomain of $T$?

   (c) What is the image of $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$?

   (d) Determine the matrix $A$ such that $T(\vec{x}) = A\vec{x}$.

   (e) Is $T$ one-to-one? Briefly explain.

   (f) Is $T$ onto? Briefly explain.

2. Provide a brief written answer to the following.

   (a) The matrix equation $A\vec{x} = \vec{b}$ is inconsistent if and only if $\text{rref}(\begin{bmatrix} A \\ \vec{b} \end{bmatrix})$.

   (b) What is the definition of the span of a set of vectors?

   (c) If a set of vectors is linearly dependent, then what does that mean? NOT how can you tell, what does it mean?

   (d) How can you tell if a set of vectors is linearly dependent?

   (e) Give one statement that is equivalent to: “Let $A$ be an $m \times n$ matrix. The matrix equation $A\vec{x} = \vec{b}$ has a solution for every $\vec{b}$ in $\mathbb{R}^m$. “