Math 106: Review for Exam I

1. Find the following. [Substitution tip: usually let \( u \) = a function that’s “inside” another function, especially if \( du \) (possibly off by a multiplying constant) is also present in the integrand.]

   (a) \( \int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx \)

   (b) \( \int_{\pi}^{2\pi} \cos^7(5x) \sin(5x) \, dx \)

   (c) \( \int \frac{7x^2}{1 + x^6} \, dx \)

   (d) \( \int \cos^3(5x) \, dx \)

   (e) \( \int_6^{10} x\sqrt{10 - x} \, dx \)

2. If \( f(x) \) is decreasing and concave up, put the following quantities in ascending order. 

   \( L_{100}, R_{100}, T_{100}, M_{100}, \int_a^b f(x) \, dx \)

   What can you say with certainty about where \( S_{200} \) would fit into your list above? [8:00 and 9:30 sections may omit this part.]
3. Suppose \( f(t) \) is the rate of change (in animals per month) of a population \( P(t) \).

(a) What does \( \int_{4}^{12} f(t) \, dt \) represent in this problem?

(b) Find the best possible left, right, midpoint, trapezoidal, and Simpson’s approximations to \( \int_{4}^{12} f(t) \, dt \) given the data in the table below. [8:00 and 9:30 sections may omit Simpson’s approximation.]

<table>
<thead>
<tr>
<th>( t )</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(t) )</td>
<td>15</td>
<td>11</td>
<td>8</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

4. Find bounds for each of the following errors if \( I = \int_{2}^{7} \ln x \, dx \).

(a) \(|I - L_{100}|\)

(b) \(|I - T_{100}|\)

(c) \(|I - M_{100}|\)

5. Use Euler’s method with three steps on the differential equation \( \frac{dy}{dt} = y - t \) to estimate \( y(2.5) \) if \( y(1) = 0 \).

6. Solve the differential equation \( \frac{dy}{dx} = 2xy + 6x \) if the solution passes through \((0, 5)\).
7. Write integrals equal to
   
   (a) the arc length of \( y = x^2 \) on the interval \([1, 5]\)

   (b) the area bounded by \( y = x^2 - 8x + 24 \) and \( y = 3x \)

8. Consider the region bounded by \( y = \sqrt{x}, y = 0, \) and \( x = 9 \). Write an integral equal to the volume generated if this region is rotated about
   
   (a) the \( x \)-axis

   (b) the line \( x = -1 \)

9. A pyramid has a square base 30 feet to a side and a height of 10 feet. Write integrals equal to
   
   (a) the volume of the pyramid \([8:00 \text{ and } 9:30 \text{ sections may omit this part, though much of it will be repeated in part(b).}])

   (b) the work done in pumping all the fluid to a point 5 feet above the pyramid if the pyramid is filled to a height of 8 feet with water (62.4 pounds per cubic foot)