MATH 105A,C - CALCULUS I  
FALL 2010  
QUIZ 2

NAME:

Show ALL your work CAREFULLY.

The graph of the derivative function $F'$ is shown below over the interval $(0, 10)$.

![Graph of $F'$](image)

(a) For what values of $x$ is the function $F$ increasing?

The function $F$ is increasing when $F'$ is positive, i.e., for all $x$ such that $0 < x < 1$ or $3 < x < 10$.

(b) Find, if any, all local maximum points of the function $F$.

At a local maximum, $F'$ must change sign from positive to negative so $F$ has a local max at $x = 1$.

(c) Find, if any, all local minimum points of the function $F$.

Similar to (b), $F$ has a local min at $x = 3$ as $F'$ changes sign from negative to positive.

(d) Find, if any, all inflection points of the function $F$.

At any inflection point, $F''$ must be 0. In addition $F''$ must change signs. Note that at $x = 2, 6, 9$, the slopes of $F'$ are zero AND the slopes change signs so these are the inflection points of $F$.

(e) For what values of $x$ is the function $F$ concave up?

Date: September 24, 2010.
The function $F$ is concave up when the second derivative $F''$ is positive, or equivalently, when the first derivative $F'$ is increasing. From the graph of $F'$, $F'$ is increasing for $2 < x < 6$ and for $9 < x < 10$.

(f) Suppose that $F(3) = 1$. Arrange the following numbers $F(3), F''(7), F'(9), F(9)$ in increasing order. Justify your answer.

From the graph of $F'$, we know that $F'(9) = 0$. $F''(7)$ is negative because it is the slope of the tangent to the graph of $F'$ at $x = 7$. We know $F(3) = 1$, our assumption. Finally, $F(9) > F(3)$ since $F$ is increasing over the interval $(3, 10)$ from part (a). Thus, we have in increasing order the numbers $F''(7), F'(9), F(3), F(9)$.

(g) Estimate the value $F''(4)$.

$F''(4)$ represents the slope of the line tangent to the graph of $F'$ at $x = 4$. From the graph above, the graph over the interval $[3, 5]$ appears to be straight so we have $F''(4) \approx \frac{F'(5) - F'(3)}{5 - 3} = \frac{2 - 0}{5 - 3} = 1$. 