1. Given the initial value problem \( \frac{dy}{dx} = -\frac{y^2}{x} \) and \( y(1) = 1 \). Euler’s method using step size \( \Delta x = 0.25 \) produces the estimate \( y(2) \approx 0.5234 \). The graph shows a plot of the points compared to the curve for the exact solution.

Recall: \( x_{n+1} = x_n + \Delta x \) and \( y_{n+1} = y_n + \Delta y \) where \( \Delta y = \text{slope at } (x_n, y_n) \cdot \Delta x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>1.25</th>
<th>1.5</th>
<th>1.75</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y' )</td>
<td>-1</td>
<td>-0.45</td>
<td>-0.2709</td>
<td>-0.1855</td>
<td></td>
</tr>
<tr>
<td>( y )</td>
<td>1</td>
<td>0.75</td>
<td>0.6375</td>
<td>0.5698</td>
<td>0.5234</td>
</tr>
</tbody>
</table>

2. Find the area of the region enclosed by the graphs of \( x = y^2 \) and \( x = -2y^2 + 3 \).

To find intersection points, solve \( y^2 = -2y^2 + 3 \) for \( y \).
\[
 y^2 = -2y^2 + 3 \iff 3y^2 - 3 = 0 \iff y = \pm 1.
\]
Therefore, the intersection points are \((1, 1)\) and \((1, -1)\).

Set up the integral in terms of \( y \) to get:
\[
\int_{-1}^{1} ((-2y^2 + 3) - y^2) \, dy = \int_{-1}^{1} (3 - 3y^2) \, dy = \left[ 3y - y^3 \right]_{-1}^{1} = 2 - (-2) = 4
\]

Remark: to set up the integral in terms of \( x \), the region needs to be separated into two pieces. The total area is given by the following:
\[
2 \int_{0}^{1} \sqrt{x} \, dx + 2 \int_{1}^{3} \sqrt{\frac{3-x}{2}} \, dx
\]
Evaluate the second integral by substitution: \( u = \frac{3-x}{2} \) therefore \( du = -\frac{1}{2} \, dx \iff -2 \, du = dx \).
To change the limits of integration: if \( x = 1 \) then \( u = 1 \). Likewise, if \( x = 3 \), then \( u = 0 \).
\[
2 \int_{0}^{1} \sqrt{x} \, dx + 2 \int_{1}^{3} \sqrt{\frac{3-x}{2}} \, dx = 2 \int_{0}^{1} x^{1/2} \, dx - 4 \int_{1}^{0} u^{1/2} \, du = 2 \left[ \frac{2}{3} x^{3/2} \right]_{0}^{1} - 4 \left[ \frac{2}{3} u^{3/2} \right]_{1}^{0} = \frac{4}{3} + \frac{8}{3} = 4
\]