Directions: Complete all of the following to the best of your ability. If you do not understand a question, please let me know; I may be able to assist you. Show all work, unless directed otherwise. You will usually be graded primarily on the method you use, not your final answer. GOOD LUCK!

All quizzes will be graded out of 10 possible points. The possible points for each question is in parentheses after the question number.

1. (6) Make some quick approximation calculations for \( I = \int_1^9 \sqrt{3x + 1} \, dx \). Write your answer in the space provided. Only your answer will be graded.

(a) What is \( x_k \) for \( n = 10? \)

(b) Fill in ALL the missing pieces around the \( \sum \) for the \( T_{10} \) approximation of \( I \). DO NOT FIND THE ACTUAL APPROXIMATION!

\[
\sum_{k=1}^{10} \Delta x [0.5f(x_k) + 0.5f(x_{k-1})] = \sum_{k=1}^{10} 0.8 \left[ 0.5\sqrt{3(1 + 0.8k)} + 1 + 0.5\sqrt{3(1 + 0.8(k - 1))} + 1 \right]
\]

2. (4) Suppose you are investigating the definite integral \( I = \int_1^4 f(x) \, dx \) for some function \( f(x) \).

(a) Suppose you learn that \( f(x) \) is increasing and concave down on the interval \([1, 4]\). Will \( R_{10} \) be an overestimate, an underestimate, or is it impossible to tell? Describe your thought process with a sentence or two!

Generally, we know that \( R_{10} \) is an overestimate for increasing functions. The fact that \( f(x) \) is concave down has no bearing in this example.

(b) After further investigation you suspect that \( f(x) = \ln(x^2 + 1) \). Find a value for \( K_1 \) needed in the error approximation of \( R_{10} \) on the interval \([1, 4]\). Describe your thought process with a sentence or two! Use the back of the quiz if you need more room.

First find \( f'(x) = \frac{1}{x} [\ln(x^2 + 1)] = \frac{1}{x^2 + 1} \cdot 2x = \frac{2x}{x^2 + 1} \). We must try to find a maximum value for \( f'(x) \) on the interval \([1, 4]\). By graphing (or even by finding \( f''(x) \)) we can determine that \( f'(x) \) achieves a maximum at the left endpoint; \( x = 1 \). So \( |f'(x)| \leq \frac{2 \cdot 1}{12 + 1} = \frac{2}{2} = 1 = K_1 \) is a value that will work.