1. Use the method of substitution to compute \( \int_1^4 \frac{e^{1 + \sqrt{x}}}{\sqrt{x}} \, dx \).

(You may either leave your answer simplified with \( e \)'s in it or you can give a decimal approximation.)

\[
\begin{align*}
\text{Let } u &= 1 + \sqrt{x} \\
\Rightarrow \quad du &= \frac{1}{2} x^{-1/2} \, dx \\
\int_1^4 \frac{e^{1 + \sqrt{x}}}{\sqrt{x}} \, dx &= 2 \int e^u \, du = 2 e^u + C = 2 e^{1 + \sqrt{x}} + C \\
\int_1^4 \frac{e^{1 + \sqrt{x}}}{\sqrt{x}} \, dx &= 2 e^{1 + \sqrt{x}} \bigg|_1^4 = 2e^3 - 2e^2 \approx 25.39 
\end{align*}
\]

2. Let \( I = \int_a^b f(x) \, dx \), where \( f \) is positive and concave up over the interval \([a, b]\). Indicate whether, for all \( n \geq 1 \), the statement must be true, cannot be true, or may be true.

(a) \( R_n \leq I \)  

(2a) \underline{may be true}  

\[\text{In (III) below } R_n \leq I \text{ but in (II) below } R_n \geq I\]

(b) \( T_n \leq I \)  

(2b) \underline{cannot be true}  

Our theorem tells us that if \( f \) is concave up, \( T_n \) must be an over estimate.

\[\begin{array}{ccc}
\text{possible } f \text{ graphs: for } f \text{ positive and concave up} \\
(\text{I}) & (\text{II}) & (\text{III}) \text{ OVER}
\end{array}\]
3. The graph below depicts the velocity of a bike (in mph). The distance traveled by the bike from time $t = 1$ to time $t = 7$ can be computed by calculating $\int_1^7 v(t) \, dt$.

*Use the Trapezoid Rule with 4 intervals (i.e., $n = 4$) to estimate the distance traveled from time $t = 1$ to time $t = 7$, i.e., to estimate $\int_1^7 v(t) \, dt$. 

$$53.25 \text{ miles}$$

\[ \Delta x = \frac{7-1}{4} = \frac{6}{4} = \frac{3}{2} = 1.5 \]

\[ T_4 = \frac{1}{2} \left[ 7.5 + 2(9.5) + 2(3.5) + 2(18.5) + 18.5 \right] 
= 53.25 \]