NAME:

Show ALL your work CAREFULLY.

(a) Consider the following given data of a function $h(x)$ on the interval $[3, 5]$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>3.0</th>
<th>3.25</th>
<th>3.5</th>
<th>3.75</th>
<th>4.0</th>
<th>4.25</th>
<th>4.5</th>
<th>4.75</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h(x)$</td>
<td>2</td>
<td>3</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>-2</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

Find $M_4$ (mid-point). Here the subscript $n$ indicates that the interval $[3, 5]$ is to be divided into $n$ equal subintervals.

When dividing the interval $[3, 5]$ into four equal subintervals, each subinterval has length $\Delta x = 0.5$. The mid-points of the four subintervals are $3.25, 3.75, 4.25, 4.75$, respectively. It follows that

$$M_4 = [h(3.25) + h(3.75) + h(4.25) + h(4.75)] \cdot \Delta x$$

$$= [3 + 0 + 1 + 0] \cdot (0.5) = 2.$$

(b) Consider the region bounded by the $y$-axis and the curves $y = 2^{x+1}$ and $y = x^3$. **SET UP (but do not evaluate)** the definite integral that represents the area of this region. [Hint: Sketch the region; find the points of intersection; set up the definite integral.]

The region lies in the first quadrant and the point of intersection between the curves $y = 2^{x+1}$ and $y = x^3$ is $(2, 8)$. By using “vertical slices”, a typical “slice” is a rectangle of area $[2^{x+1} - x^3] \cdot \Delta x$. It follows that the area of this region is given by

$$\int_0^2 2^{x+1} - x^3 \, dx.$$