Math 105 - Quiz 2 - September 14, 2007

Instructions: Show all of your work and circle your final answers. Calculators are allowed, but notes and books are not.

1. (7 pts.) Show that \( f(x) = \frac{3x}{x+2} + \frac{4}{5x+6} \) is a rational function. (For this problem, it is not enough to just say that the sum of two rational functions is a rational function.)

\[
f(x) = \frac{3x}{x+2} + \frac{4}{5x+6} = \frac{3x(5x+6)}{(x+2)(5x+6)} + \frac{4(x+2)}{(5x+6)(x+2)}
\]

\[
= \frac{3x(5x+6) + 4(x+2)}{(x+2)(5x+6)}
\]

\[
= \frac{15x^2 + 18x + 4x + 8}{5x^2 + 10x + 12}
\]

\[
= \frac{15x^2 + 22x + 8}{5x^2 + 10x + 12} \quad \text{This is a polynomial divided by a polynomial, so it is a rational function.}
\]

2. (7 pts.) Suppose the function \( T \) has period 6. If \( U(x) = T(5x) \), what is the period of \( U \)? Explain.

Graphically, \( T \) repeats itself every 6 units.

Since \( U(x) = T(5x) \), the graph of \( U \) is the graph of \( T \) compressed horizontally by a factor of 5. So, \( U \) repeats itself every \( \frac{6}{5} = 1.2 \) units, so the period of \( U \) is \( 1.2 \).

3. (6 pts.) Find the range of \( g(x) = 4 \sin(2x) \).

Range of \( \sin x = [-1, 1] \).

\( \sin(2x) \) is horizontally compressed so it has the same range.

\( 4 \sin(2x) \) is stretched vertically by factor of 4.

So its range is \( [-4, 4] \).