Math 105B Final Exam

- There are 8 questions
- You have 2 hours

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Name:
1. (3 marks) Evaluate the following limits without a calculator.

a. \( \lim_{x \to -2} \frac{2x^4 + 4x^2 - 8x - 16}{x + 2} \)

b. \( \lim_{x \to 0} \frac{\sin x}{\sqrt{x}} \)

c. \( \lim_{x \to \infty} \frac{x^2 + 3}{2x - 5} \)
2. (3 marks) Find $\frac{dy}{dx}$.

a. $y = \frac{x^2}{2} - \frac{1}{x} + \arcsin x$

b. $y = x^3 \ln x - \cos(\sqrt{x}) + \frac{\ln x}{2x}$

c. $y = \int_a^{2x} f(t) \, dt$
3. (3 marks) Find antiderivatives for the following functions.

a. \( f(x) = x^5 + \sec^2 x + \sin(2x) + \ln 2 \)

b. \( f(x) = \frac{1}{2\sqrt{x}} + \frac{1}{1+x^2} + e^x \cos x + e^x \sin x \)

c. \( f(x) = \frac{\sin x}{(3+\cos x)^2} \)
4. (3 marks) Evaluate the following integrals.

a. \[ \int_{0}^{\pi} \sin x + e^x \, dx \]

b. \[ \int_{1}^{4} 5x + x^3 \, dx \]

c. \[ \int_{0}^{1} \frac{2x}{x^2 + 1} \, dx \]
5. (3 marks) Write the following sums in sigma notation.

a. \(a_1b_5 + a_2b_4 + a_3b_3 + a_4b_2 + a_5b_1\)

b. \(1 + r + \frac{r^2}{2} + \frac{r^3}{3!} + \frac{r^4}{4!} + \frac{r^5}{5!}\)

c. \(1 + 3 + 5 + 7 + 9\)
6. (6 marks) Airline A only allows carry on bags whose longest side plus its girth (girth=distance around the box NOT using the longest side) does not exceed 112 inches. What are the dimensions of the bag with two square ends that has largest volume?
7. (6 marks) A 5 foot 4 inch woman is walking towards a 12 foot tall streetlight at a rate of 2 feet per second. When she is 12 feet from the base of the light, at what rate is the length of her shadow changing?
8. (3 marks) Pick one.

Prove the product rule using the limit definition of derivative.

OR

Prove that if \( f \) is a continuous bounded function on the set of real numbers then the graph of \( f \) crosses the line \( y = x \) at least once.