Math 106: Review for Final Exam, Part I

1. Find the following. [See Review for Exam II for integration tips and strategies.]

(a) \( \int 12x^2 \cos(x^3) \, dx \)

(b) \( \int_0^\infty xe^{-3x} \, dx \)

(c) \( \int_0^6 \frac{dx}{(x-4)^2} \)

(d) \( \int 3x^2 + 2x - 5 \frac{dx}{(x^2 + 1)(x - 4)} \)

(e) \( \int_0^{\pi/3} \tan^3 x \sec^5 x \, dx \)
2. Find the best possible left, right, midpoint, trapezoidal, and Simpson’s approximations to \( \int_{-2}^{0} f(x) \, dx \) given the data in the table below.

<table>
<thead>
<tr>
<th></th>
<th>-2</th>
<th>-1.5</th>
<th>-1</th>
<th>-0.5</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>

3. If you use numerical integration to estimate \( \int_{a}^{b} \ln x \, dx \) (where \( a \) and \( b \) are positive), how would the following be ordered from least to greatest? \( L_{100}, R_{100}, M_{100}, T_{100}, \int_{a}^{b} \ln x \, dx \).
4. Find bounds for each of the following errors if \( I = \int_0^2 e^{-5x} \, dx \).

   (a) \( |I - R_{100}| \)

   (b) \( |I - T_{100}| \)

   (c) \( |I - M_{100}| \)

5. Write an integral equal to the area between \( y = 2x + 3 \) and \( y = x^2 + 7x - 3 \).

6. Compute the arc length of \( y = \sqrt{1 - x^2} \) from \( x = 0 \) to \( x = 1/2 \).

7. Consider the region bounded by \( y = 0 \), \( x = 2 \), and \( y = x^2 \). Write an integral equal to the volume of the object created when the region is revolved about

   (a) the \( x \)-axis

   (b) the line \( x = 5 \)
8. A spherical tank of radius 8 feet is buried 5 feet below ground and filled to a height of 11 feet with gasoline (42 pounds per cubic foot). Write an integral equal to the work done in pumping all the gasoline to ground level. [Students in the 1:10 section should omit this problem.]

9. The probability density function (pdf) of the weights of newborn toads in a certain pond is given by

\[ f(x) = \frac{k}{(x + 1)^4}, \] where \( x \) is the weight (in ounces). Note that the domain is \( x \geq 0 \) since no toad can have a negative weight.

(a) What must be the value of \( k \)?

(b) What fraction of the newborn toads weigh more than one ounce?

10. Find the solution to \( \frac{dy}{dx} = \frac{\cos x}{y^2} \) that passes through \( (0, 2) \).