1. Recall that the Taylor series for $e^x$ centered at $x_0 = 0$ is $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$ on $(-\infty, \infty)$.

(a) (5 pts.) Find the first four non-zero terms of the Taylor series for $f(x) = xe^{-2x}$ centered at $x_0 = 0$.

(b) (5 pts.) What is $f^{(13)}(0)$? Justify your answer.

2. (10 pts.) Consider the initial value problem $\frac{dy}{dx} = x + y$ and $y(2) = 1$. Apply Euler’s method using step size $\Delta x = 0.25$ to estimate $y(3)$. Round to 4 digits after the decimal. (Note: you are not required to use the table, but regardless of your method, you need to show enough work to justify your answer.)

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>1</td>
</tr>
<tr>
<td>$\frac{dy}{dx}$</td>
<td></td>
</tr>
<tr>
<td>$\Delta y$</td>
<td></td>
</tr>
</tbody>
</table>
3. (8 pts each) Evaluate the following integrals.

(a) \( \int_0^1 \arcsin x \, dx \)

(b) \( \int \frac{\sqrt{x^2 - 9}}{x} \, dx \)
(c) \[ \int \frac{x^3}{x^2 - 1} \, dx \]
4. (10 pts.) Find the interval of convergence for the series \( \sum_{k=1}^{\infty} \frac{3}{k^5} x^k \).

5. (10 pts.) Determine if \( \sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2} \) converges or diverges. Justify your answer.
6. (10 pts.) Use separation of variables to solve the following initial value problem. Be sure to use the initial condition to determine the value of any constant you introduce.

\[
\frac{dy}{dt} = t^2 e^y \quad \text{with} \quad y(3) = 0
\]

7. (8 pts) Consider the region bounded by \( y = 16 - x^2 \) and the \( x \)-axis. Set up, **but do not evaluate**, the integral that represents the volume of the solid found by revolving this region about the line \( y = -3 \).
8. (3 pts. each) Complete the following.

(a) The series \(1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \cdots\) converges to __________.

(b) If \(f\) is decreasing, but concave up, then when a midpoint sum is used to approximate \(\int_a^b f(x) \, dx\) the approximation will be ________________ because . . .

(c) The \(n^{th}\) term test allows us to conclude that a series of the form \(\sum_{k=1}^{\infty} a_k\) will __________ if . . .

(d) The series \(1 + x + x^2 + x^3 + x^4 + \cdots\) converges for all \(x\) such that __________. For such an \(x\) the series converges to . . .

(e) The Alternating Series Test states that . . .

(f) Error bounds for using the \(n^{th}\) order Taylor Polynomial \(P_n(x)\) to estimate \(f(x)\) on an interval \(I\) may be determined using: 
\[|f(x) - P_n(x)| \leq \frac{K}{(n+1)!}|x - x_0|^{n+1},\] where \(K\) is found using the __________ derivative such that . . .
Formulae you may find useful.

• $\int \sec \theta \, d\theta = \ln |\sec \theta + \tan \theta| + C$

• $\int \csc \theta \, d\theta = \ln |\csc \theta - \cot \theta| + C$

• $\int \cot \theta \, d\theta = -\ln |\csc \theta| + C$

• $\sum_{k=1}^{\infty} \frac{1}{k^p}$ converges for $p > 1$

• Log Properties
  - $\ln(xy) = \ln x + \ln y$
  - $\ln \frac{x}{y} = \ln x - \ln y$
  - $\ln x^y = y \ln x$

• Trigonometric Identities
  - $\sin^2 \theta + \cos^2 \theta = 1$
  - $\sec^2 \theta = 1 + \tan^2 \theta$
  - $\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$
  - $\sin(2\theta) = 2 \sin \theta \cos \theta$
  - $\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$
  - $\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$