1. Let $R$ be the region in the plane consisting of all points which are below the line $y = x + 1$ and above the parabola $y = (x - 5)^2$. The graph of $R$ is shown here.

1A. Set up and fill in all the formulas for the integral(s) in the form \( \int_a^b (\text{"right curve"} - \text{"left curve"}) \, dy \) that represent the area of this region $R$. (Do not evaluate the integral(s)).

1B. Set up the integral(s) which represent the volume of the solid of revolution obtained by revolving $R$ around the line $y = -2$. (Do not evaluate the integral(s)).
2. Consider \( I = \int_1^8 f(x) \, dx \) where \( f(x) = \frac{\sin x}{x} \). Fact: \( \frac{d}{dx} \left( \frac{\sin x}{x} \right) = \frac{\cos x}{x} - \frac{\sin x}{x^2} \).

2A: What is value of \( K_1 \) you should use in “theorem 3” to find an error bound for the approximation RHS(150) produces for \( I \)? Use an appropriate graph on your calculator to estimate \( K_1 \) correctly to two digits after the decimal point.

2B: What is that error bound in (2A)? (to five digits after the decimal point)? Show the formula you use in your work.

2C: What is RHS(150) (to five places after the decimal point)?

2D: What is the value of \( I \) as found by your calculator’s built-in numerical integration routine (under the CALC menu)? (Again to five digits after the decimal point).

2E: Here’s another fact. For all \( x \) in the interval \([1,8]\), we have \( |f''(x)| < 0.25 \). Use theorem 3 to find a value of \( N \) which guarantees that \(|I - \text{MID}(N)| < 0.001\).

2F: To five places after the decimal point, what does your calculator find for \( \text{MID}(N) \) for your value of \( N \)?

2G: How far off from the calculator’s answer in (2D) is your \( \text{MID}(N) \) in (2F), and is the difference less than the guarantee of 0.001?
3A. Use the method of substitution to find \( \int_{0}^{\pi/2} \frac{\sin x}{e^{\cos(x)} - 1} \, dx \) Hint: start by rewriting the integrand \textit{not} as a fraction (put everything in the numerator). Express your final answer in terms of \( e \) and as simply as possible. NO DECIMALS.

3B. In particular, what are the new limits on the integral after the substitution is made?

4. Use integration by parts to find \( \int_{0}^{1} 3xe^{4x} \, dx \). Show all your steps.
5. Let \( f(x) = \frac{x + \sqrt{4 - x}}{x \sqrt{4 - x}} \). If you graph \( f \) on the interval \([0, 4]\) you should see that \( f \) has an asymptote at each endpoint.

Here’s a useful fact: \( \ln(x) - 2\sqrt{4 - x} \) is an antiderivative of \( f \).

5A) Is the area under the graph of \( f \) on \([0, 2]\) bounded or unbounded? Use and evaluate an appropriate improper integral in your answer to this question. If the area is bounded, what is that area?

5B) Is the area under the graph of \( f \) on \([2, 4]\) bounded or unbounded? Again, use and evaluate an appropriate improper integral to answer this question. If the area is bounded, what is that area?
6. Consider \( g(x) = \frac{2x^3 - 17x^2 + 18x - 45}{(x^2 + 9)(x^2 - 6x + 9)} \)

6A) The partial fraction decomposition (PFD) of \( g(x) \) has \( \frac{Ax + B}{x^2 + 9} \) in it, plus what additional term(s)?

6B) It turns out that \( A = 2 \), and there will be no \( \ln |x - 3| \) term in the antiderivative of \( g(x) \) (so at least one of the other constants (maybe \( B \) or \( C \) or \( D \)? in the PFD has to be 0 and now you know which one!). With this help, find the entire PFD.

6C) Find \( \int g(x) \, dx \).
7. (A) For each of the functions below, find its Taylor series in the list that follows, and write the number of the series in the box. For example, if you think $e^x$ has series (B), you’d write “B” in the first box here.

i) $e^x$  
ii) $\sin x$  
iii) $\ln(x + 1)$  
iv) $\arctan x$

A: $x - \frac{1}{6} x^3 + \frac{1}{120} x^5 - \frac{1}{5040} x^7 + \frac{1}{362880} x^9 + \cdots$

B: $x - \frac{1}{3} x^3 + \frac{1}{5} x^5 - \frac{1}{7} x^7 + \frac{1}{9} x^9 + \cdots$

C: $1 + x + \frac{1}{2} x^2 + \frac{1}{3} x^3 + \frac{1}{4} x^4 + \frac{1}{5} x^5 + \cdots$

D: $1 - x^2 + x^4 - x^6 + x^8 - \cdots$

E: $2 \cdot 1x + 4 \cdot 2x^2 + 8 \cdot 3x^3 + 16 \cdot 4x^4 + 32 \cdot 5x^5 + \cdots$

F: $1 + x + \frac{1}{2} x^2 + \frac{1}{6} x^3 + \frac{1}{24} x^4 + \frac{1}{120} x^5 + \cdots$

G: $1 - \frac{1}{2!} x^2 + \frac{1}{4!} x^4 - \frac{1}{6!} x^6 + \frac{1}{8!} x^8 + \cdots$

H: $x - \frac{1}{2} x^2 + \frac{1}{3} x^3 - \frac{1}{4} x^4 + \frac{1}{5} x^5 - \cdots$

Z: none of the above. (If you choose this answer, give the series you are looking for in the space below).

7. (B) Let $f(x)$ be the function whose series is “E” in the above list. Show how the ratio test is used to determine the interval of convergence (except perhaps for the endpoints) for this series. Also, determine if the series converges at either endpoint.
8. (A) Consider \( \int_0^8 g(x) \, dx \) where \( g(x) = 1 + \sin \left( \frac{\pi}{4} (x - 2) \right) \). Although this integral is easy enough to evaluate by hand, to avoid mistakes, find it using your calculator’s built-in numerical method (under the CALC menu).
What’s the answer? \( \square \) (Hint! The number you should get has already appeared on this page somewhere!)

8B. Now how should you choose \( K \) so that
\[
f(x) = \begin{cases} K \left(1 + \sin \left( \frac{\pi}{4} (x - 2) \right) \right) & \text{if } 0 \leq x \leq 8 \\ 0 & \text{otherwise,} \end{cases}
\]
is a probability density function (PDF)? \( K = \square \)

8C. (Use your calculator to numerically find any integrals that you want in the following). Suppose that \( f \) is the probability density function (PDF) for the number of light bulbs that burn out at time \( x \) years. Answer these questions to four places after the decimal point:
8C i) What’s the probability that a light bulb chosen at random from this collection will burn out in the first 4 years?

8C ii) What’s the probability that a light bulb will burn out between 2 and 5 years of use?

8D. Suppose instead the light bulb lifespan has the normal distribution (call it \( b(x) \)), with mean 4 (years) and standard deviation 2 (years). Answer these using your calculator’s built in normal density function, where \( X \) is the random variable representing this light bulb lifespan.
8D i) What is \( P(0 \leq X \leq 4) \)?

8D ii) Find \( P(2 \leq X \leq 5) \).

8 BONUS 1: At the review, a classmate suggested a very practical way to find \( \int_4^\infty b(x) \, dx \) using the calculator. Find the integral by that method, to as many digits as your calculator displays. Show the calculations.

8 BONUS 2: If you graph \( f \) and the normal density function \( b(x) \) in (8D) together, you’ll see they have similar shapes. You already know what the inflection points of \( b(x) \) are. Show analytically that \( f \) has inflection points at the same two places. Are the \( y \)-coordinates the same?
9. Let \( s(x) \) be the function defined by the Taylor series
\[
s(x) = 2x - 4x^3 + 6x^5 - 8x^7 + 10x^9 - \cdots.
\]
(Its IOC is \((-1, 1)\) but you don’t have to check this).

9A. Find \( s^{(9)}(0) \). (Recall: use the formula for the coefficient \( c_k \) in a Taylor series — it involves derivatives).

9B) Find \( s^{(8)}(0) \).

9C) Find the first five non-zero terms in the series for \( s'(x) \).

9D) After the constant \( C \) of integration, what are the next five non-zero terms in the family of antiderivatives \( \int s(x) \, dx \)?

9E) Use the answer to 9D to find an approximation of \( \int_0^{0.1} s(x) \, dx \).

9F) How far off at most is the approximation in 9E from the true value of the integral? Explain! Hint: Is this an alternating series?