1. (5 points)
   Consider the graph of $f(x)$ shown at right. Use $M_3$ to estimate $\int_2^8 f(x) \, dx$.

2. (5 points) Let $I = \int_0^2 \frac{9}{130} x^{13} \, dx$. How many subdivisions $n$ are required so that the error in using $L_n$ to estimate $I$ is less than 0.005.

3. (20 points) Find the following integrals. If an integral is improper, determine whether or not it converges and find its value if it does.
   
   (a) $\int x \cos x \, dx$
   
   (b) $\int_1^2 \frac{dx}{(1 - x)^{2/3}}$
   
   (c) $\int_0^\infty \frac{x}{\sqrt{x^2 + 2}} \, dx$
   
   (d) $\int_1^2 \frac{dx}{x^3 + x}$

4. (5 points)
   The shaded region at right is bounded by the graphs of $x = -(y - 1)^2 + 3$, $y = 1 - x$, and $x = 0$. Write an expression involving integral(s) that gives the area of the shaded region. **You do not have to evaluate or simplify the integral(s).**
5. (9 points) Let \( f(x) = x^{1/3} \).
   (a) Find the second order Taylor polynomial based at \( x_0 = 8 \) for \( f(x) \).
   
   (b) Use your answer to (a) to estimate \( f(9) \).

6. (10 points) Determine whether the following series converge or diverge. **Justify your answers.** If the series converges find an exact value or give good upper and lower bounds.
   (a) \( 96 - 48 + 24 - 12 + 6 - 3 + \cdots \)
   
   (b) \( \sum_{n=2}^{\infty} \frac{2^k - 1}{3^k + 4} \)

7. (9 points) \( S = \sum_{j=1}^{\infty} \frac{(-1)^j}{\sqrt{j}} \) converges by the Alternating Series Test (you do NOT need to verify this).
   (a) Is this series absolutely convergent or conditionally convergent? Justify your answer.
   
   (b) Find good upper and lower bounds for \( S \).

8. (5 points) Find the interval of convergence of the power series: \( S(x) = \sum_{k=1}^{\infty} \frac{x^k}{3^k k^2} \).

9. (12 points) Let \( f(x) = \frac{1}{1 - 2x^3} \).
   (a) Write the first five terms of the Taylor series for \( f(x) \) based at \( x_0 = 0 \). Simplify your answer.
   
   (b) Calculate \( f^{(48)}(0) \). You do not need to simplify your answer.
   
   (c) Write the first five terms of the power series representation of \( \int_{0}^{0.5} \frac{1}{1 - 2x^3} \, dx \). You do not need to simplify your answer.

10. (18 points) Let \( g(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2n + 1} x^{2n+1} \).
    (a) Write the first five terms of the power series \( g'(x) \).
    
    (b) Which of the following functions equals \( g'(x) \)?

        (I) \( \frac{1}{1 - x^2} \)  
        (II) \( \frac{1}{1 + x^2} \)  
        (III) \( \frac{-1}{1 - x^2} \)  
        (IV) \( \frac{-1}{1 + x^2} \)

    (c) Use (b) to determine what function \( g(x) \) is. Justify your answer.
    
    (d) Find \( \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2n + 1} \) exactly.

11. (2 points) What is your favorite shape?