1. (20 points) Find the arc length. \( f(x) = x^2, \quad x \in \left[ -\frac{1}{2}, \frac{1}{2} \right] \)

2. (20 points) Find the volume of the solid of revolution formed when the region bounded by \( y = x^2 - 1 \) and \( y = x + 1 \) is revolved about the line \( y = -1 \).

3. (20 points) A conic gasoline tank is formed by rotating the segment of \( y = x, \quad x \in [0, 30] \) about the \( y \)-axis. The tank is full, and gasoline weighs 43 lb/ft\(^3\). Find the amount of work needed to pump all the gasoline in the tank to a nozzle that is 10 feet above the top of the tank.

4. (20 points) Evaluate the integral. \( \int \sin^4 x \cos^2 x \, dx \)

5. (20 points) Evaluate the integral. \( \int \frac{dx}{(x^2 - 25)^{3/2}} \)

6. (20 points) Evaluate the integral. \( \int \frac{dx}{x(x + \sqrt{2})} \)

7. (20 points) Determine whether the integral is convergent or divergent. If it converges, evaluate it. If it diverges, give reasons. \( \int_0^e x^2 \ln x \, dx \)

8. (20 points) Determine whether the series are convergent or divergent. If they converge, find their sums.

a) \( \sum_{n=1}^{\infty} \left( \cos \frac{1}{n} - \cos \frac{1}{n+1} \right) \)  
   b) \( \sum_{n=0}^{\infty} \frac{(\ln \pi)^n}{n!} \)  
   c) \( \sum_{n=0}^{\infty} \frac{(\cos \pi)^n}{(\ln \pi)^n} \)

9. (20 points) Determine whether the series are absolutely convergent, conditionally convergent, or divergent.

a) \( \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n+1}} \)  
   b) \( \sum_{n=0}^{\infty} \frac{\sqrt{n^2 - 1}}{n^3 + 2n^2 + 5} \)

10. (20 points) Determine the radius and interval of convergence of the series.

\( \sum_{n=1}^{\infty} \frac{(3x - 2)^n}{n3^n} \)

11. (20 points) Find a power series representation for the function. \( f(x) = \frac{2x}{(1 + x)^3} \)

12. (20 points) Determine whether the series \( \sum_{n=1}^{\infty} a_n \) is absolutely convergent, conditionally convergent, or divergent.

a) \( a_1 = 2, \quad a_{n+1} = \frac{5a_{n+1} + 3}{4a_{n+3}} a_n \)  
   b) \( a_1 = 1, \quad a_{n+1} = \frac{2 + \cos n}{\sqrt{n}} a_n \)