1.) (10 pts.) Given the quadratic form $8x_1^2 + 6x_1x_2$,

   a.) (2 pts.) find the symmetric matrix of the quadratic form;

   b.) (2 pts.) classify the quadratic form as positive definite, negative definite, or indefinite, and explain your reasoning;

   c.) (6 pts.) make a change of variable, $x = Py$, that transforms the quadratic form into one with no cross-product term.
2.) (15 pts.)

a.) (5 pts.) **True or False:** A least-squares solution of $Ax = b$ is a vector $\hat{x}$ such that $\|b - Ax\| \leq \|b - A\hat{x}\|$ for all $x$ in $\mathbb{R}^n$. If this is true, explain why. If it is false, correct the statement to make it true.

b.) (10 pts.) While boiling a pot of water, you take the temperature every two minutes. This generates the data points $(0, 15), (2, 37), (4, 68), (6, 89)$, where the first coordinate is time, in minutes, and the second coordinate is temperature, in degrees Celsius. Using linear algebra techniques, find the equation $y = \beta_0 + \beta_1 x$ of the least-squares line that best fits these data points.
3.) (15 pts.)

a.) (5 pts.) Is it true that \( \mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{u} = 0 \) for every pair of vectors \( \mathbf{u} \) and \( \mathbf{v} \) in \( \mathbb{R}^n \)? If so, explain why; if not, explain why not.

b.) (5 pts.) Suppose both \( U \) and \( V \) are orthogonal matrices. Explain why \( UV \) is an orthogonal matrix. That is, explain why \( UV \) is invertible and why its inverse is \((UV)^T\).

c.) (5 pts.) The Orthogonal Decomposition Theorem gives a formula for computing \( \hat{y} \), the projection of a vector \( y \) onto a subspace \( W \) of \( \mathbb{R}^n \). Each term in that formula is itself a projection of \( y \) onto a subspace of \( W \). Explain why.
4.) (15 pts.)

a.) (5 pts.) Verify that \( \mathbf{v} = (2, 1, -1, 2) \) is an eigenvector of \( A \), given below. What is the corresponding eigenvalue of \( \mathbf{v} \)?

\[
A = \begin{bmatrix}
-6 & 4 & 0 & 9 \\
-3 & 0 & 1 & 6 \\
-1 & -2 & 1 & 0 \\
-4 & 4 & 0 & 7
\end{bmatrix}
\]

b.) (5 pts.) Construct a 4 \( \times \) 4 matrix with eigenvalues \(-3, 2, \) and 5 (with multiplicity 2). Your matrix should not be strictly diagonal - that is, there must be some nonzero entries in non-diagonal positions within the matrix.

c.) (5 pts.) Use the factorization \( A = PD P^{-1} \) to compute \( A^k \), where \( k \) represents an arbitrary positive integer.

\[
A = \begin{bmatrix}
33 & -20 \\
60 & -37
\end{bmatrix} = \begin{bmatrix}
2 & 1 \\
3 & 2
\end{bmatrix} \begin{bmatrix}
3 & 0 \\
0 & -7
\end{bmatrix} \begin{bmatrix}
2 & -1 \\
-3 & 2
\end{bmatrix} = PD P^{-1}
\]
5.) (15 pts.)

a.) (5 pts.) What are the three properties of a subspace $H$ of $\mathbb{R}^n$?

b.) (5 pts.) The shaded region in the image below is a set in $\mathbb{R}^2$. (Include the bounding lines as part of the set.) Give a specific reason why this set is not a subspace of $\mathbb{R}^2$.

c.) (5 pts.) Let $A$ be an $m \times n$ matrix. Explain why $\text{Nul} \ A$ is a subspace of $\mathbb{R}^m$. 
6.) (15 pts.)

a.) (5 pts.) Must an elementary matrix be square? Why or why not?

b.) (5 pts.) Let $T$ be the linear transformation $T(x_1, x_2, x_3) = (4x_2 - 6x_3, 0, 7x_2 - 9x_3, x_1)$. Find the matrix $A$ for which $T(x) = Ax$.

c.) (5 pts.) Use the matrix inverse algorithm to compute $A^{-1}$, if it exists. If it does not exist, explain how the algorithm shows this.

$A = \begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$
7.) (15 pts.)

a.) (5 pts.) Suppose the vectors below are linearly independent. What can you say about the numbers \( a, b, c, d, e, \) and \( f \)?

\[
\begin{bmatrix}
  a \\
  0 \\
\end{bmatrix}, \quad 
\begin{bmatrix}
  b \\
  c \\
  0 \\
\end{bmatrix}, \quad 
\begin{bmatrix}
  d \\
  e \\
  f \\
\end{bmatrix}
\]

b.) (5 pts.) Write the coefficient matrix of the system of equations below.

\[
\begin{align*}
3x_2 &- 6x_3 + 8x_4 = -5 \\
3x_1 &+ x_3 - 2x_4 = 7 \\
4x_1 &+ x_2 + 5x_3 = 8
\end{align*}
\]

c.) (5 pts.) Write the augmented matrix of the system of equations in part (b). Does the system have a solution? How do you know?