NAME:

Instruction: Read each question carefully. Explain ALL your work and give reasons to support your answers.

Advice: DON’T spend too much time on a single problem.

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1. Evaluate each of the following integrals.

(9 pts.) (a) 
\[
\int \frac{x}{(1 + x^2)^{3/2}} \, dx 
\]

(9 pts.) (b) 
\[
\int \frac{\ln x}{x^2} \, dx 
\]
2. Evaluate each of the following integrals.

(9 pts.) (a) \[
\int \frac{x - 10}{2x^2 - 5x - 3} \, dx
\]

(9 pts.) (b) \[
\int \frac{dx}{(x^2 + 4)^{3/2}}.
\]
(10 pts.) (a) Use the method of separation of variables to solve the following Initial Value Problem.

\[ \sqrt{x} + \sqrt{y} \frac{dy}{dx} = 0, \quad y(1) = 9. \]

(b) Consider the following given data of a function \( h(x) \) on the interval \([-3, 0]\).

\[
\begin{array}{c|cccccc}
 x & -3 & -2.5 & -2 & -1.5 & -1 & -0.5 & 0 \\
 \hline
 h(x) & -1 & 2 & 0 & -1 & 3 & 2 & -4 \\
\end{array}
\]

Find \( M_3 \) (midpoint sum) and \( L_6 \) (left-hand sum).
4. (9 pts.) (a) Let $R$ be the region bound by the curves $y = \sqrt{x}$ and $y = \frac{x^2}{8}$. Find the exact area of the region $R$. [First sketch a picture of the region.]

(13 pts.) (b) Find the exact volume of the solid formed when the region $R$ (in part (a)) is revolved around the $y$-axis.
5. (8 pts.) (a) Evaluate the following improper integral if it exists.

\[ \int_{0}^{2} \frac{1}{\sqrt{x-1}} \, dx \]

(6 pts.) (b) Write a definite integral [Do Not Evaluate] representing the arc length of the graph of

\[ f(x) = \ln(\cos x) \]

over the interval \([0, \frac{\pi}{4}]\). Simplify the definite integral as much as possible.
6. Determine whether each of the following series converges or diverges. **Justify your answer.**

(8 pts.) (a) \[ \sum_{n=0}^{\infty} \frac{2n - 1}{3n + \sqrt{n} + 1} \]

(8 pts.) (b) \[ \sum_{j=1}^{\infty} \frac{j}{\sqrt{j^4 + 3j}} \]

(8 pts.) (c) \[ \sum_{k=1}^{\infty} \frac{(2k)!}{k^3} \]
7. (10 pts.) (a) Let $f(x) = \arctan x$. Use the Maclaurin series for $\arctan x$ to find $f^{(50)}(0)$ and $f^{(71)}(0)$. [Hint: what is $\frac{d}{dx} \arctan x$?]

(8 pts.) (b) Find a series representation of $\frac{\sin(x^2)}{x}$. [Be sure to state the interval of convergence.]
8. (10 pts.) (a) Find the radius and the interval of convergence for the following power series. [Don’t forget to check the endpoints.]

\[ \sum_{n=1}^{\infty} \frac{(x - 4)^n}{n5^n} \]

(8 pts.) (b) Determine whether the following series converges absolutely, conditionally, or neither.

\[ \sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n} \]