Math 105: Review for Final Exam, Part II

1. Consider the function \( f(x) = x^3 \ln x \) on the interval \([1/e, e^2]\).
   
   (a) Find the \( x \)- and \( y \)-coordinates of any and all local extrema and classify each as a local maximum or local minimum.

   (b) Find the \( x \)- and \( y \)-coordinates of any and all global extrema and classify each as a global maximum or global minimum.

   (c) Find the \( x \)-coordinate(s) of any and all inflection points.

2. You are watching a plane flying toward your position at a constant height of 3 miles and a speed of 500 miles per hour relative to the ground. At the moment when the plane is 5 miles from you (diagonally), at what rate is the angle of your vision toward the plane changing?
3. Your company is mass-producing a cylindrical container. The flat portion (top and bottom) costs 3 cents per square inch and the curved (lateral) portion costs 5 cents per square inch. If your budget is $9.00 per container, what dimensions will give the largest volume?

area of circle = \( \pi r^2 \)  

lateral area of cylinder = \( 2\pi rh \)  

volume of cylinder = \( \pi r^2 h \)

4. For the following questions, suppose that the graph shown is \( f'(x) \) and that \( f(1) = -4 \).

(a) If \( g(x) = f(x)/e^x \), is \( g \) increasing, decreasing or stationary at \( x = 1 \)?

(b) If \( h(x) = f(\sin x) \), is \( h \) concave up, concave down, or neither at \( x = 0 \)?

5. Use the Intermediate Value Theorem to show that \( f(x) = x^3 - x + 1 \) has a root on \([-2, 0]\).
6. What (if anything) does the Extreme Value Theorem say about \( f(x) = x^2 \) on each of the following intervals?
   (a) \([1, 4]\)
   (b) \((1, 4)\)

7. Find the value of the constant \(c\) that the Mean Value Theorem specifies for \( f(x) = x^3 + x \) on \([0, 3]\).

8. Find the following.
   (a) \(\int_1^7 \frac{3}{x} \, dx\)
   (b) \(\int_1^4 (1 + 2x + x^3 + 4\sqrt{x} + \frac{1}{x^5}) \, dx\)
   (c) \(\int_{-2}^{2} \sqrt{4 - x^2} \, dx\)
   (d) \(\frac{d}{dx} \int_1^x \sin \sqrt{t} \, dt\)
   (e) \(\lim_{n \to \infty} \frac{2}{n} \sum_{k=1}^{n} \left(1 + \frac{2k}{n}\right)^2\)

9. Water is leaking out of a tank at a decreasing rate \(r(t)\) as shown in the table below.

<table>
<thead>
<tr>
<th>time (min)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>rate (gal/min)</td>
<td>15</td>
<td>11</td>
<td>8</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

(a) Find an overestimate and underestimate for the total amount that leaked out during these 8 minutes.

(b) Interpret the expression \(\int_2^6 r(t) \, dt\) in terms of the situation described above.
10. Consider the graph of $f(t)$ shown. It is made of straight lines and a semicircle.

Let $G(x) = \int_0^x f(t) \, dt$ and $H(x) = \int_{-3}^x f(t) \, dt$.

(a) Compute $G(2)$, $G(4)$, and $H(4)$.

(b) Where is $G$ increasing? Where is $G$ decreasing?

(c) Where is $G$ concave up? Where is $G$ concave down?

(d) At what $x$-value(s) does $G$ have a local maximum? At what $x$-value(s) does $G$ have a local minimum?

(e) Find a formula that relates $G$ and $H$.

(f) How would your answers to (b), (c), and (d) change if the questions were about $H$ instead of $G$?

11. (a) Use sigma notation to express $L_{10}$ and $M_{10}$ as approximations to $\int_{20}^{60} \ln x \, dx$.

(b) Draw a sketch that represents the sum $M_4$. 