Read all of the following information before starting the exam:

- Put all of your work in the blue book EXCEPT for the last problem (do that on the test sheet). Turn in the blue book and test sheet.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 10 problems and is worth 100 points, It is your responsibility to make sure that you have all of the pages!
- Good luck!
- Some useful formulas:
  Area of a circle with radius \( r \), \( A = \pi r^2 \),
  Circumference of a circle with radius \( r \), \( C = 2\pi r \).
  Area of a rectangle with sides \( x \) and \( y \), \( A = xy \),
  Perimeter of a rectangle with sides \( x, y \), \( P = 2x + 2y \)
  Volume of a rectangular prism with sides \( x, y, z \), \( V = xyz \),
  Surface area of rectangular prism with \( x, y, z \), \( SA = 2xy + 2yz + 2xz \)
  Volume of a cylinder with radius \( r \) and height \( h \), \( V = \pi r^2 h \),
  Surface area of a cylinder with radius \( r \) and height \( h \), \( SA = 2\pi r^2 + 2\pi rh \)

\[
\sum_{i=1}^{n} i = \frac{n(n+1)}{2}
\]
\[
\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}
\]
\[
\sum_{i=1}^{n} i^3 = \left( \frac{n(n+1)}{2} \right)^2
\]
1. (6 points) Sketch a possible graph of a single function $f$ that satisfies all the following conditions:

1. $f'(x) > 0$ on $(-\infty, 1)$, $f'(x) < 0$ on $(1, \infty)$
2. $f''(x) > 0$ on $(-\infty, -2)$ and $(2, \infty)$, $f''(x) < 0$ on $(-2, 2)$
3. $\lim_{x \to -\infty} f(x) = -2, \lim_{x \to \infty} f(x) = 0$

2. (19 points) Find the derivative using the appropriate rules.
   
   a. (6 pts) $h(x) = (\cos^3(x) + 3\cos(x) + 7)^9 + \frac{\sqrt{x+1}}{x+2} + 2e^{-x^2/2}$

   b. (5 pts) $y = \frac{\sin^4(x)e^{3x}}{(4x^4 - 3x^2 - 2)^5}$ using logarithmic differentiation.

   Use the following table to find the derivatives of the functions at the given values of $x$.

   \[
   \begin{array}{c|ccc}
   x & 1 & 2 & 4 \\
   \hline
   f(x) & 4 & 2 & 6 \\
   f'(x) & 5 & 7 & 4 \\
   g(x) & 4 & 1 & 6 \\
   g'(x) & 5 & 1 & 3 \\
   \end{array}
   \]

   c. (4 pts) $g(\sqrt{x})f(x)$ at $x = 4$.

   d. (4 pts) $f(g(x) - 2x)$ at $x = 1$.

3. (8 points) Water pours into a fish tank (standard rectangular prism) at a rate of $3 \text{ ft}^3/\text{min}$. How fast is the water level rising if the base of the tank is a rectangle with dimensions $2 \text{ ft} \times 3 \text{ ft}$?
4. **(10 points)** A warehouse consists of three separate spaces of equal size. Assume that the wall materials cost $200 per linear foot and the company has allocated $2,400,000 for the project.

   ![Warehouse Diagram]

   a. **(8 pts)** What dimensions maximize the total area of the warehouse?

   b. **(2 pts)** What is the area of each compartment in this case?

5. **(12 points)** Use L’Hopital’s Rule if applicable. If not, use any other algebraic method learned this semester. You may certainly check your answer with a table.

   a. **(4 pts)** \[
   \lim_{x \to 16} \frac{\sqrt{x} + 4}{x - 16}
   \]

   b. **(4 pts)** \[
   \lim_{x \to 0} \frac{e^x - x - 1}{\cos x - 1}
   \]

   c. **(4 pts)** \[
   \lim_{x \to \infty} x^{1/x}
   \]

6. **(8 points)** Let \( y \) be a function of \( x \). Use implicit differentiation to find the an equation of the tangent line at the point \((1,1)\) on the curve

   \[ y^4 + xy = x^3 - x + 2 \]

7. **(7 points)** Determine the antiderivative of \( e^{6t} + \frac{2t^3}{2 + 8t^4} + \frac{2}{2 + 8t^2} \).

8. **(8 points)** Verify the applicability of the IVT in the indicated interval for the give value. IF APPLICABLE find a value of \( c \) guaranteed by the theorem. IF NOT APPLICABLE, explain.

   a. **(4 pts)** \( f(x) = x^2 + x - 1, [-2, 5], f(c) = 11 \)

   b. **(4 pts)** \( f(x) = x^2 + x - 1, [-2, 5], f(c) = 0 \)

9. **(14 points)** Let \( f(x) = 3x^2 - 4x \) on \([0,3]\).

   a. **(2 pts)** Find \( L_3 \).

   b. **(4 pts)** Determine \( \int_0^3 (3x^2 - 4x)dx \) using the FTC.

   c. **(8 pts)** Use infinite Riemann sums to find the area under the curve \( f(x) = 3x^2 - 4x \) on \([0,3]\).
Let $F$ be an antiderivative of $f$. Consider the following proof that $F(b) - F(a) = \int_a^b f(x)dx$. Fill in the blanks. (***) should be the same entry.

\[
F(b) - F(b) = F(x_n) - F(x_0)
\]

Using telescoping sums, $F(x_n) - F(x_0)$ can be written as

\[
\sum_{i=1}^{n} f(x^*_i) \Delta x
\]

We can write this in summation notation,

\[
(**) \sum_{i=1}^{n} \text{__________________}.
\]

Since $F(x)$ is continuous and bounded on $[a, b]$ and differentiable on $(a, b)$, the MVT guarantees that there exists an $x^*_i$ in $[x_{i-1}, x_i]$ such that

\[
\text{__________________}.
\]

which we can rewrite as

\[
\text{__________________}.
\]

Then by substitution,

\[
(**) \sum_{i=1}^{n} \text{__________________} = \sum_{i=1}^{n} \text{__________________}
\]

Since,

\[
\text{__________________}.
\]

the sum is the same as $\sum_{i=1}^{n} f(x^*_i) \Delta x$.

If we take the limit as $n$ approaches $\infty$ of both sides, then

\[
F(b) - F(a) = \lim_{n \to \infty} \sum_{i=1}^{n} f(x^*_i) \Delta x = \int_a^b f(x)dx.
\]